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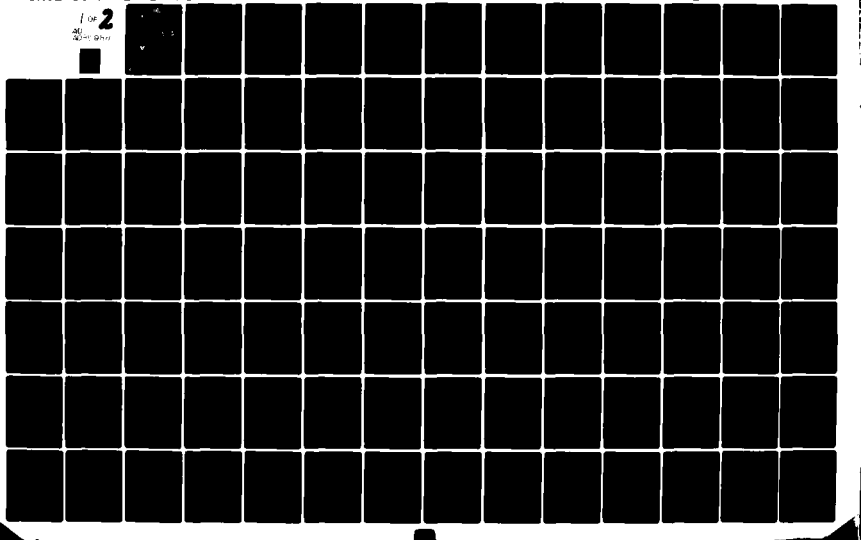

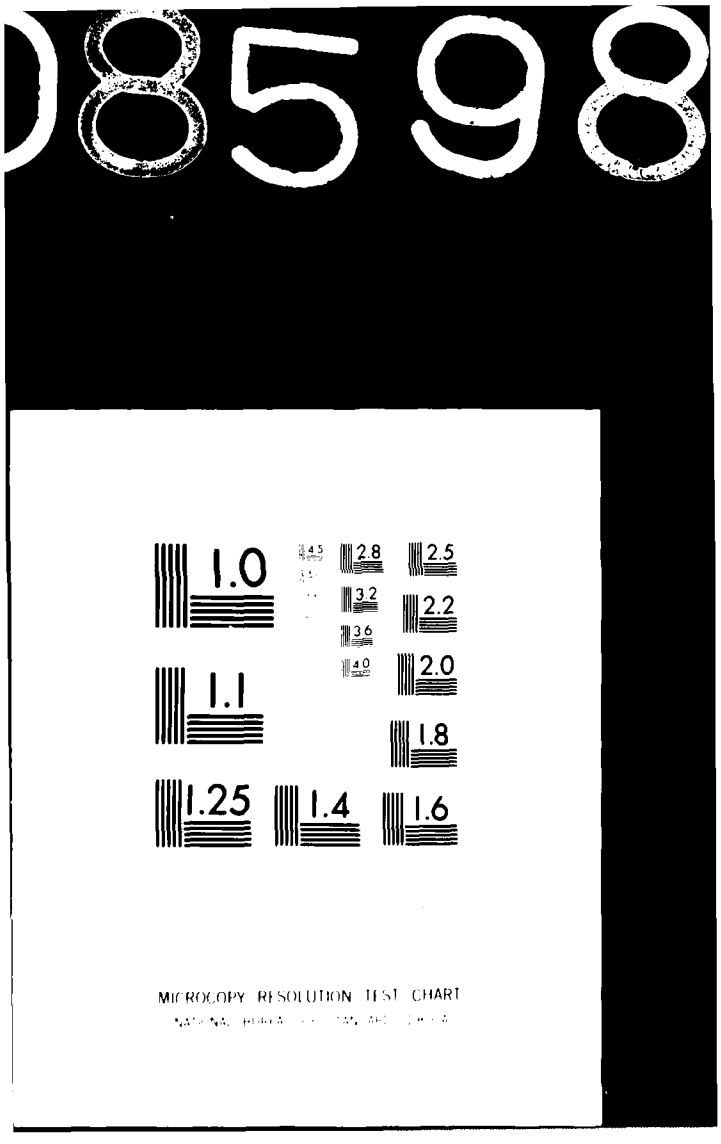
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RESEARCH AND DEVELOPMENT PROJECT PRIORITIZATION – AN ANNOTATED BIBLIOGRAPHY

Edward B. Dobbins, Jr.
Technology Integration Office
US Army Missile Laboratory

April 1980

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1. REPORT NUMBER: Management Report RN-80-1	2. GOVT ACCESSION NO.: AD A085988
3. RECIPIENT'S CATALOG NUMBER: (9)	
4. TITLE *(and Subtitle)*: (6) RESEARCH AND DEVELOPMENT PROJECT PRIORITIZATION, AN ANNOTATED BIBLIOGRAPHY.	5. TYPE OF REPORT & PERIOD COVERED: Management Report (13)
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): (10) Edward B. Dobbins, Jr	8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Commander, US Army Missile Command, Attn: DRSMI-RN, Redstone Arsenal, AL 35809	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS: Commander, US Army Missile Command, Attn: DRSMI-RPT, Redstone Arsenal, AL 35809 (11)	12. REPORT DATE: April 80 (12) 97 13. NUMBER OF PAGES: 91
14. MONITORING AGENCY NAME & ADDRESS *(if different from Controlling Office)*	15. SECURITY CLASS. *(of this report)*: UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT *(of this Report)*

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT *(of the abstract entered in Block 20, if different from Report)*

(14) DRSMI/RN-80-1

18. SUPPLEMENTARY NOTES

19. KEY WORDS *(Continue on reverse side if necessary and identify by block number)*

Aggregation	Majority Rule
Rank Orders	Circular Triads
Research and Development	Transivity
Prioritization	Social Welfare Function
Fuzzy Set Rank Orders	Social Choice

20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

This report is based upon research concerned with the aggregation of multiple lists of rank-ordered research and development (R&D) projects or product requirements needing R&D. The resultant prioritized list serves as a basis for resource allocation to the R&D projects. The rank orders are ordinal and without feedback or strategy. The extensive literature of this field of study, social choice, is structured and analyzed in this report.

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20. ABSTRACT (Continued)

A companion document, an Industrial Engineering Management Dissertation by Dobbins, builds upon the literature foundation to classify the known majority-rule aggregation methods, evaluate them, and develop a model and computer code using the better methods to compute the aggregation of up to 100 rank-ordered lists of up to 100 alternatives. Final rank-ordered priority lists are computed by the preferred Shannon method and compared to the BORDA and fuzzy set rank-order methods.

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I. INTRODUCTION

This report begins with a statement of the problem and a need discussion followed by the scope of the research effort.

A. Statement of the Problem

The problem addressed in this report and Dobbins [1] is to develop and demonstrate a methodology, with its associated computer model that will acceptably transform several individual multicriteria rank-ordered lists of research and development (R&D) projects into a single aggregated, prioritized rank-ordered list to guide the investment of R&D sources. The methodology developed must be capable of aggregating, with reasonably small effort, very long individual partial length and/or full length lists.

B. Need for this Research Solution

The task of R&D management planning for high technology systems has become more difficult. The emphasis on coordination and communication between the management groups of major functional elements of high technology systems developmental organizations, both industrial and governmental, and the expanded usage of goal and objective planning methods have complicated the planning process. Situations have resulted where the planners in the R&D element receive many diverse priority lists of suggested future R&D work or products from the other functional elements (i.e., marketing, field operations, production, and senior staffs) and from managers within the R&D organization. The prioritization criteria of interest to each contributing element differ as their functions and objectives differ. Therefore, the individual prioritizations are based to varying extent, upon the objective criteria and viewpoints of each element. The R&D element management must combine these lists into a usable list of prioritized R&D projects for their consideration in the allocation of discretionary R&D funds for use toward advancing the technology base of the enterprise.

C. Scope of this Research

The research documented by this report is a comprehensive literature survey of the subjects of social choice and majority-rule methods that are applicable to the aggregation, without feedback, of multiple criteria rank-ordered ordinal priorities. From this basis, the research will determine and develop the specific majority-rule methodology to aggregate the variety of rank-ordered priority lists, as described previously, for R&D project priority determination.

II. SURVEY OF PAST RELATED LITERATURE

This section contains the results of a comprehensive survey of the literature related to the research topic. The section contains a chronological and network analysis of the literature thrusts as well as synopses of key material. An extensive annotated bibliography was prepared and is published in Section III.

A. Overview of the Literature

Extensive available literature was surveyed to select and evaluate the "Social Choice" relevant body of knowledge and its application to the problem of the aggregation of several ordinal R&D project preference rank orders into a single rank-ordered list. This final rank order should provide a reasonable representation of the consensus of the individual preferences. Since the known literature was primarily in the fields of welfare economics, political science, and social science, it had to be interpreted and translated to determine its relevance to R&D engineering management. For a better understanding of the material and for the application of the available knowledge, the relevant literature was separated into distinct thrust areas of emphasis. These thrust areas then were interrelated on a time axis to determine when and where the later thrusts branched off from the initial endeavors.

The search of the extensive relevant literature did not identify any work dealing directly with the goal of this research. There were no majority-rule type methods for aggregation of multi-criteria, ordinal rank-ordered lists to obtain a single rank-order list of R&D projects for resource investment. A few articles did recognize that R&D project selection was a possible application for social choice theory.

Certain restrictions were used to select the relevant material from the extensive body of social choice information. These restrictions include the following:

1) The rank orders of interest are ordinal, not cardinal, preferences.

2) The emphasis is on the aggregation of individual ranked preferences into single group rank-order lists.

3) The rank ordering and aggregation are single cycle, not temporal, decisions.

4) There is no feedback from individuals to other group members.

5) The individuals who rank apply their sincere beliefs and are not using strategy to coerce the group result to agree with their preferences or objectives.

The literature selected was grouped into 25 thrust area groups of interest and of manageable size. Briefly, these thrust area groups contain:

a) Pre-Arrow Basic-The basic theory in the social choice and majority-rule areas before the publication of Kenneth Arrow's Impossibility Theorem.

b) Statistical Correlation Methods-The works that developed methodologies for the determination of the correlation measures between multiple ordinal rank orders.

c) Single-Peakedness-The research which developed the single-peakedness concept originated by Duncan Black.

d) Majority-Rule Models-The theory development and description of the many aggregation rules which use a majority-decision rule. (This area contains papers that discuss single methods or single aspects of methods.)

e) Number of Voters and Alternatives-Research which develops and quantifies the point where the numbers of voters and alternatives severely decrease the probability of a majority-rule method providing a usable aggregate rank-order.

f) Arrow's Impossibility Theorem-The original works of Kenneth Arrow and the other researchers who clarified and critiqued his significant Impossibility Theorem.

g) Extensions and Revisions of Arrow's Impossibility Theorem-The works of researchers who offered approaches to extend or revise Arrow's Theorem either for clarification, claimed corrections, or improved usefulness through the determination of limiting conditions which will permit the possibility of obtaining a social welfare function.

h) General Social Choice Theory Overview-Works, mostly books, which broadly cover the field of social choice theory.

i) Social Welfare Function-The works which further Arrow's efforts to define a mathematical function which will describe the alternative preferences preferred by a free society.

j) Comparisons of Borda and Condorcet-Works which compare the majority-rule methods developed by these two eighteenth century French scientists.

k) Majority Rule-Multiple Methods Compared-Works that describe and comparatively analyze several majority-rule methods.

l) Transitivity, Intransitivity, and Cyclicity-Works which develop the theory and special conditions to permit majority-rule aggregation methods to generate usable, transitive rank orders.

m) Basic Arrow and Majority-Rule Theory-Works which expand upon the theoretical base of Arrow's work and of majority-rule methods.

n) Majority Rule-Minimum Loss Methods-Research which develops majority-rule methods that utilize the concepts of consensus optimizing by minimizing loss to the individuals.

o) Survey Literature-Works which broadly survey the literature in the social choice field.

p) Majority-Rule-Game Theory and Strategy Proofness-The research which approaches majority-rule methods through game theory techniques. This area also includes one game theory paper on a majority rule that is intended to be strategy proof.

q) Tullock's Books-The controversial works and critiques of the works of Gordon Tullock.

r) Majority-Rule Examples-Works which describe examples of the application of majority-rule methods.

s) Vote.'s Paradox-Works which describe and analyze the voter's paradox (intransitivity) which can occur in an aggregation of rank orders.

t) Majority-Rule Conditions and Equilibrium-Works which offer conditions for better, usually transient, majority-rule results.

u) Weighting Methods-Works describing mathematical methods to apply weights to rank-ordered individual lists, or to select alternatives in those rank orders to adjust for perceived inequities.

v) Majority-Rule-Graphical Methods-Works which approach majority-rule theory through graphical methods.

w) Resource Allocation by Voting-Research which described the allocation of resources by voting techniques.

x) Majority-Rule-Minimum Distance Technique-Works which develop majority-rule methods that utilize concepts of minimization of the distance between each alternative in the rank orders.

y) Fuzzy Set Rank Orders-Works which describe fuzzy set mathematical methods to develop rank orders from matrix elements.

The reference material was chronologically sequenced within each thrust area. To obtain an indication of the time span of work in each area, the date of the earliest, the median, and the latest paper in each group is recorded in Table 1. Also listed is the number of papers and books relevant to this dissertation that were grouped in each area. The first area, a) Pre-Arrow Basic, has four works that span from 1953 translations of the 1770 papers to 1958 with a median paper dated 1953. The last area, y) Fuzzy Set Rank Ordering, has ten papers which span from 1974 to 1979 with a 1978 median date. Except for two areas, Pre-Arrow Basic and Tullock's Books, all of the work areas appear to be active at the writing of this dissertation and more material can be reasonably expected to be published.

Figure 1 presents a network visualization of the interrelation along a time axis, of the 25 literature thrust areas. More specialized areas branch off from d) Majority-Rule Models thrust. The o) Tullock's Book's activity starts and ends within the time scale of this network. The network begins to branch rapidly during the period immediately after the publications of Arrow's Impossibility Theorem. The current continuation of work in 23 of the 25 areas and the relatively late median dates listed in Table 1, reflect the accelerating activities of the 1970's, possibly kindled by the several excellent books published since the mid-1960's by authors such as Sen and Fishburn.

Over the last three centuries certain authors, in the judgment of this writer, have significantly contributed to the body of knowledge most relevant to the work for this dissertation. In the eighteenth century Borda and Condorcet developed two majority-rule methods of such merit that the Borda method is still used today and the Condorcet methods are frequent goal references for comparisons with modern techniques for aggregation of rank orders. In the nineteenth century, Dodgson, who is better known as Lewis Carroll, the author of Alice's Adventures in Wonderland (1865), developed another worthy majority-rule method. The twentieth century contributors began with the rank-order statistics and aggregation methodologies developed by Kendall between the 1930's and the 1950's. Black's single-peakedness technique and historical documentation of early writers in the 1940's and 1950's laid a firm foundation for the future expansion of work led by Arrow's Impossibility Theorem in 1951.

Two leading authors of the 1960's, Sen and Pattanaik, published thorough theoretical books on Social Choice.

Copeland, in an unpublished paper in 1951, developed a majority-rule, two-step method that has been highly praised in comparative analyses. Svestka and Shannon developed a useful two-step majority-rule method similar to Copeland's method. In the 1970's, two outstanding authors were Fishburn, for this diverse and thorough writings across the Social Choice field, and Richelson, who produced thorough

TABLE 1. LITERATURE TOPIC GROUPS

Categories	Publication			
	First	Median	Last	No.
A. Pre-Arrow Basic	(1770)	1953	1958	4
B. Statistical Correlation Methods	1939	1956	1977	11
C. Single-Peakedness	1948	1975	1976	7
D. Majority Rule Models	1948	1973	1978	18
E. Number of Voters and Alternatives	1948	1976	1979	7
F. Arrows Impossibility Theorem	1951	1972	1978	14
G. Extensions and Revisions of Arrow's Impossibility Theorem	1952	1972	1978	23
H. General Social Choice Theory Overviews	1953	1973	1978	9
I. Social Welfare Function	1953	1974	1979	9
J. Comparisons of Borda and Condorcet	1953	1975	1977	20
K. Majority Rule-Multiple Methods	1954	1971	1978	8
L. Transitivity, Intransitivity, and Cyclicity	1954	1972	1978	19
M. Basic Arrow and Majority Rule Theory	1956	1974	1978	11
N. Majority Rule-Mi.. Loss Methods	1960	1963	1976	4
O. Survey Literature	1961	1973	1978	15
P. Majority Rule - Game Theory and Strategy Proofness	1961	1975	1979	9
Q. Tullock's Books	1962	1969	1973	6
R. Majority Rule Examples	1963	1970	1975	3
S. Voter's Paradox	1965	1968	1976	7

TABLE 1. (CONCLUDED)

Categories	Publication			
	First	Median	Last	No.
T. Majority Rule Conditions and Equilibrium	1966	1973	1979	15
U. Weighting Methods	1968	1972	1975	6
V. Majority Rule - Graphical Methods	1968	1974	1976	4
W. Resource Allocation	1970	1971	1975	3
X. Majority Rule - Minimum Distance	1973	1974	1978	3
Y. Fuzzy Set Rank Ordering	1974	1978	1979	10

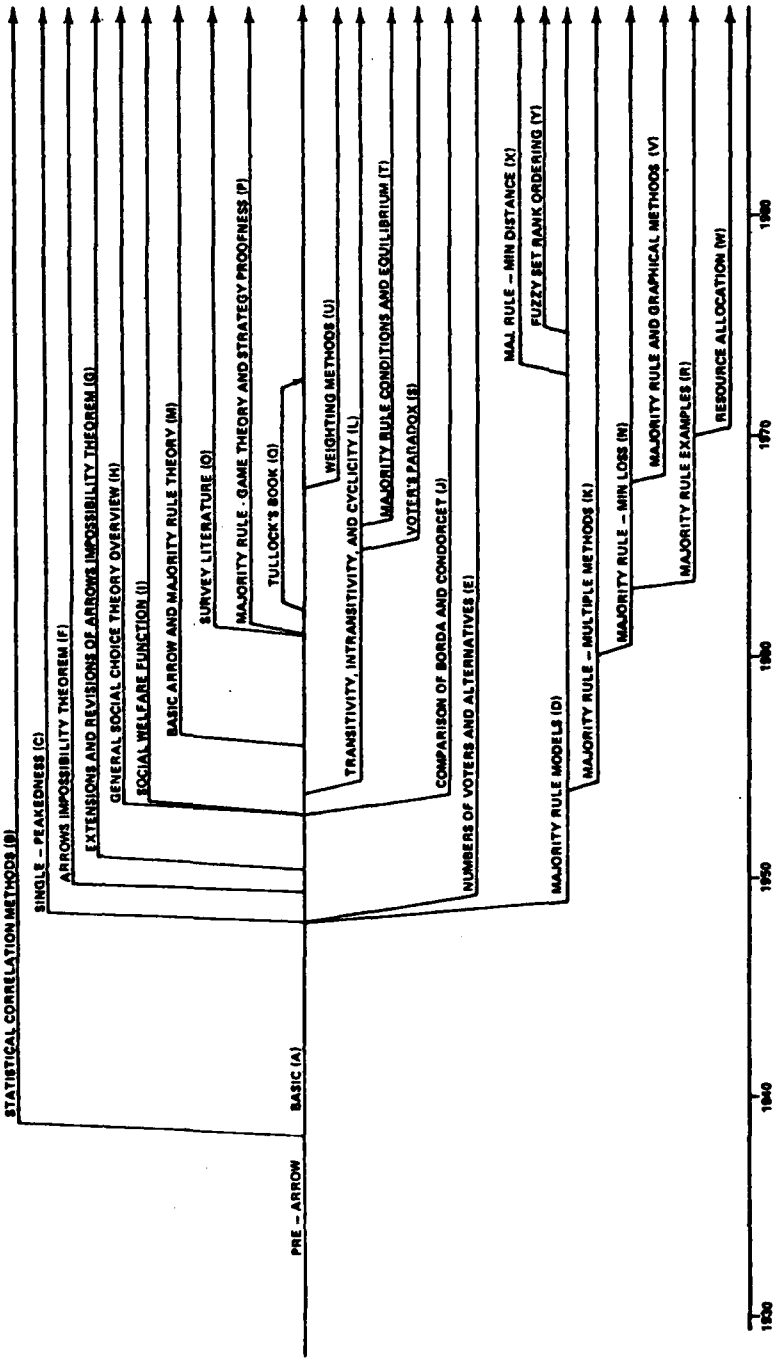

Figure 1. Individual preference aggregation literature growth tree.

characteristics comparisons, evaluations of majority-rule methods, and the classification of majority-rule conditions.

Cook's promising works, which are in process, apply Industrial Engineering optimization techniques to majority-rule aggregation methods. This chronological treatment of major authors, limited by the space available and this writer's experience and judgment, is only intended to be a representation of the hundreds of excellent researchers who have contributed, and still are contributing, to the social choice field.

B. Synopses of Literature

The following paragraphs will give a brief synopsis of over 200 books and articles selected as relevant to the research. Each book and article is annotated in somewhat more detail in Chapter III. All are listed and cross-referenced in the Bibliography. The material presented in each area of the synopses and in Section III of this report has been listed chronologically according to the publication year. The synopses are as follows:

1) Pre-Arrow Basic: Methods to use the preferences of the majority of voters to obtain a single consensus apparently were recorded [2] first in 1770 when Borda recommended his "method of marks." Condorcet's true majority and LaPlace's method followed closely after Borda's work. Dodgson (Lewis Carroll) recommended modifications to Borda's method during the mid-1800's. The economic interest in rank-order aggregation was kindled through Robbins' [3] 1932 contention, supported further by Bergson [4], that the magnitudes of individual preferences cannot be added but must be analyzed by ordinal means. (A)

2) Statistical Correlation Methods: Kendall, Smith, and Friedman [5, 6, 7], developed the statistical rank-ordered correlations, such as the coefficient of concordance method. (B)

3) Single-Peakedness: Black's contention [8, 9, 10] that single-peaked orders had preferred aggregation characteristics was substantiated and expanded upon by others [11, 12, 13]. (C)

4) Majority-Rule Methods: Various majority-rule methods have been developed either as derivatives of the classical methods (Borda, Condorcet, etc.) or as new approaches. These include a vote score assignment equation by Schuler [14], an aspiration level overlay by Harnett [15], a majority-rule Kendall-derived two-matrix method by Shannon [16], a point system by Smith [17], a branch and bound algorithm method by Cook [18], and a conversion from ordinal to cardinal rank orders, before aggregation, method by Wood and Wilson [19]. (D)

5) Number of Voters and Alternatives: Fishburn [20, 21, 22] and Bell [23] developed several papers that quantify the likelihood that

a simple majority method will produce a winner as the quantity of voters and of alternatives vary. Dutta and Pattaniak [24] and Greenberg [25] added theoretical depth to these data. (E)

6) Arrow's Impossibility Theorem: K. Arrow, in 1951, determined [26, 27] the required conditions for a social welfare function which aggregates individual preferences, then proved the theorem that a fair social welfare function, without a dictatorship, was impossible. The required conditions were more simply and clearly stated by Little [28] as follows:

a) Retrievability of alternatives.

b) An alternative's relative position in an individual order will not relatively change in the aggregate order.

c) The independence of irrelevant alternatives.

d) Non-imposition.

e) Non-dictatorship.

Numerous researchers countered his Theorem [29, 30], offered modifications [31, 32, 33], or presented alternatives proofs [34, 35, 36] of Arrow's results. (F)

7) Extensions and Revisions of Arrow's Impossibility Theorem: While striving to overcome the "impossible," social science researchers developed new or modified conditions for social welfare functions that could be satisfied, albeit from a reduced variety of acceptable individual orders. May [37] defined conditions of decisiveness, symmetry, neutrality, and positive responsiveness. Other conditions defined are summation of ranks [38], unanimity and monotonicity [39], split groups of indifferent alternatives [40], intensity added to the independence of irrelevant alternatives [41], index of degree of intensity of antagonism, [42], anonymity [43], group rationality [44]. Others reaffirmed Arrow's Impossibility results through different assumptions [45, 46, 47, 48, 49]. (G)

8) General Social Choice Theory Overview: As knowledge developed, several comprehensive social choice theory works became available from authors such as Luce and Raiffa [50], Patternaik [51, 52], Fishburn [53], Herzbuger [54] and Ferijohn and Page [55]. (H)

9) Social Welfare Function: The definitions and conditions for social welfare functions have been analyzed [56, 57, 58, 59, 60, 61], while the question of the existence of such functions has been debated [62, 63]. (I)

10) Comparison of Borda and Condorcet: In recent years many researchers have comparatively analyzed the majority-rule methods developed by Borda and Condorcet. De Grazia [2] and Black [10] provided translations and analyses of the original papers. Fishburn [64, 65], Young [66, 67] and Richelson [68, 69, 70] compared the two methods. The Condorcet method was the reference method of several papers [65, 71, 72], while Borda's method was used for other [73, 74, 58, 65, 75, 76, 77]. (J)

11) Majority Rule-Multiple Methods Compared: Goodman described [197] the Copeland two-step method and compared it favorably to other methods, as did Richelson [68, 69, 70]. Svestka [78], Wyatt [79], Chariter and Wertheimer [80], and Castore, Peterson, and Goodrich [81] made comprehensive comparisons of lists of majority rule and other aggregation methods. (K)

12) Transitivity, Intransitivity, and Cyclicity: An accepted limitation of the unrestricted use of many majority-rule methods is the real probability of aggregated results which are intransitive or cyclic. Many researchers have worked to better understand the characteristics and causes of transivity, intransitivity, and cyclicity, and to develop conditional restrictions that will control these aspects of rank-order aggregation. May [82] and others strived to clarify intransivity [83, 84, 85, 86], while Inada and others [76, 98, 116, 139, 152, 157, 167, 199, 210, 216] developed decision rules to avoid intransitive results, generally through restrictions on the characteristics of the individual rank orders. (L)

13) Basic Arrow and Majority Rule Theory: New axiomatic structures for preference theory were developed by Luce [87], Fishburn [88, 89, 90], and Grandmont [91], while Koopman [92] clarified concepts in existing theory. (M)

14) Majority Rule-Minimum Loss Methods: Van den Boggard and Versluis [93] and others [94, 95, 96] developed techniques to aggregate individual welfare preferences by minimization of the social loss function which was based upon individual loss functions. (N)

15) Survey Literature: Surveys of the work and publishings of others in the field were made by Coombs and Riker [97, 98], Guilbaud [214], Sen [100], Fishburn [53], Plott and others [101, 102, 103, 104, 216]. (O)

16) Majority Rule-Game Theory and Strategy Proofness: Based upon the 1947 work of von Neumann and Morgenstern [186], game theory has been applied to the simple majority concept. Barbut [105] introduced the relation of the two-person, zero-sum game to majority rule methods, while Shisko [106] extends this approach for n-person majority-rule games. The relevant theory was developed and extended [107, 108, 109]. Gibbard [110] applied the game theory approach to develop the criteria for a strategy-proof voting scheme.

17) Tullocks Books: G. Tullock stated in his books [111, 112] that Arrow's Impossibility Theorem would seldom be important and that little had been contributed in the majority-rule area since Black developed single-peakedness. Tullock's critical writings generated responses from Arrow [113] and others [114, 115]. (Q)

18) Majority Rule Examples: A few reports were found that represented practical examples of the application of the majority rule [116, 117, 118]. (R)

19) Voter's Paradox: Effort has continued to generalize the analysis to determine the probability of achieving an intransitive aggregate rank order through a majority-rule method [119, 120, 121, 122, 123, 124]. (S)

20) Majority Rule Conditions and Equilibrium-Diverse efforts have continued striving to determine the conditions required for a usable majority-rule decision. Murakami [125] determined that the scoring constants should logically be (1, 0, -1), have autonomy, be non-reversed and have non-dictatorship. Pattanaik [126] pursued the value restrictions techniques, while Fine [127] recommended monotonicity and faithfulness. Blau and Deb [128] proved that an infinite social decision function does not produce a choice. Richelson [129] classified and showed the interrelations of the multitude of overlapping conditions for a social choice function. (T)

21) Weighting Methods: Weighting permits special importance to be considered for certain judges' rankings or for certain alternatives. Winkler [130] and others [131, 132] summarized most useful methods: equal weights, weights proportional to ranking, weights proportional to self-rating, and weights based on previous assessments. Gustafson, Pai, and Kramer [133] reported on a weighting method for hierarchical R&D project selection. Rowse, Gustafson, and Ludke [134] added peer weights, group weights, and average weights to the list of available methods. (U)

22) Majority Rule-Graphical Methods: Research has continued toward describing the social decision process through graphical structures [94, 135, 136, 137]. (V)

23) Resource Allocation by Voting: Several studies were made of the effect of voting rules on resource allocation processes such as capital budgeting and R&D project selection [138, 139, 140]. (W)

24) Majority Rule-Minimum Distance Technique: Bowman and Colantoni [141] developed a majority-rule rank-order decision method based upon the minimizing of a decision function defined as a one-dimensional "distance." The scoring constants used are (1, 1/2, 0) and the distance function is $d(P^*, \overline{P}) = f[\Sigma_{ij} d_{ij}(P^*_{ij}, \overline{P}_{ij})]$. Blin

and Whinston [142] suggest a quadratic assignment solution to the Bowman and Colantoni method. Cook and Seiford [143] extended the one-dimensional distance minimization method so that it can be solved by linear programming techniques. (X)

25) Fuzzy Set Rank Ordering: Blin originally proposed [144, 145] that fuzzy binary relations could be used for group preference orderings. Others [146, 147, 148, 149, 232], especially Bezdek, Spillman, and Spillman [150, 151] have developed Blin's proposition into a new area of social choice study. Buckles [152] class work in translating from Fuzzy Set terminology to that of majority rule has been beneficial for this research. (Y)

III. DISCUSSION

This section contains a summarizing discussion of the reference literature chosen for this research. The material is grouped into topic thrust areas as described in Section II. Within each group, the material is in chronological order. The Bibliography contains the normal citation information and a cross reference back to the sections of this report.

A. Pre-Arrow Basic

In 1953, Alfred de Grazia [2] included an English translation of Jean-Charles de Borda's 1770 paper: "Memoir On Election By Ballot." Borda's stated purpose was to design a pure and just majority system. Borda developed a method where votes select the order of merit of all candidates. Then the number of votes a candidate receives for each merit place (first, second, third, etc.) is multiplied by that position's number, i.e., multiply by 0 for last place, by 1 for next to last place, etc. Then the products are summed to give a result to denote the candidate's aggregated position. Borda comparatively demonstrated that the plurality method of summing each candidate's number of first place votes resulted in an inferior winner to that of the order of his merit method. His method gives each candidate a "mark" equal to the sum of the votes that he would get when he is matched against each of the other candidates individually. This method was the beginning of the binary type majority rule aggregation schemes that have evolved since 1770.

In 1958, Duncan Black [8] provided detailed reviews of the history of mathematical theory of committees and elections. Black explained the works of Jean-Charles de Borda (1733-1799), Marquis de Condorcet (1743-1794), Marquis de Laplace (1749-1827), and several English mathematical theoreticians expecially Rev. C. L. Dodgson (1832-1898), who also wrote literature under the name of Lewis Carroll. Black explained the Borda method, then proceeded to relate it to the method of Condorcet. Black explained that only the outline, not the clear

recipe, of the Cordorcet method was understood. Cordorcet's general method is to examine the case of three candidates and then extend the results to n candidates. If one of three candidates is able to get a majority over the others, he ought to be elected. He uses a simple rule when there are three propositions out of more than three alternatives, with majorities in their favor: delete the one with the lowest majority and take the straightforward interpretation of the other two. Though not explicit, Black explains several interpretations of Condorcet's method. Later authors, making assumptions, present specific methods they attribute to Condorcet. These will be explained later.

Black described the method of Laplace as one of a voter giving each candidate's measurable merit, i.e., the highest merit value for the most meritorious candidate, less merit for the second candidate, etc. The candidate who is elected is the one to whom the most merit is attributed by the group of voters. This scheme is based upon the merit attributed, through interpersonal comparison, by one elector to be exactly the same kind as that attributed by another.

Black attributed C. L. Dodgson (Lewis Carroll) with recommending a modification of the method of marks (a Borda method) which treats the possibility of "no election" as if it were the name of a candidate, and assigns marks so as to discourage the voter from "bracketing candidates together."

In 1932, Robbins [3] presented the concept that the aggregation of personal, or social, preferences can be ordered and compared with the orders of others. But he stated and tenaciously defended in his 1935 second edition that this does not imply that there are magnitudes that relate to the orderings. In summary, he contended that preference aggregations should be analyzed by ordinal methods, not by cardinal methods. For the economist, he contended that there is no way of comparing the satisfaction levels of different people. This position and his expansion thereof became a solid basis for forthcoming ordinal analysis of social welfare and social utility.

In 1938, Bergson [4] contended, in his paper defining welfare economics, the proposition that ordinal value analysis must be used in the welfare calculus instead of the cardinal utility calculus that had been introduced by other economists. He further stated that value propositions which imply complete measurability of economic welfare functions is not necessary to welfare economic analysis.

B. Statistical Correlation Methods

The driving question in statistically analyzing multiple ordinal rank orders is to determine a measure of the agreement between complete rank orders and tests to determine the significance of the agreement's measurements. Several authors, especially Kendall and

Friedman, developed measures of rank correlation. Several of these books and articles are highlighted in the following paragraphs.

In 1939, Kendall and Smith [7] developed, with examples, the coefficient of concordance, W, parameter to indicate the agreement between sources of rank orders (sources can be individual judges or groups of judges). Further, the paper describes the statistical tests to measure the significance of W.

In 1940, Friedman [6] compared the χ^2 statistic and the Kendall-Smith W, which is defined as χ_r^2, divided by its maximum value, m(n-1), where n is the number of alternative ranks and m is the number of rank orders tested. Friedman documents and explains both methods and compares their application. He concludes that although the Kendall-Smith is more accurate, the χ_r^2 statistic is adequate where it applies.

In 1947, Moran [153] extended the Kendall-Smith number of circular triads computations to the case where the pairs are indifferent. He determines the distribution for d, the number of triads, when there is a chance of indifference. He concludes the distribution tends to be normal.

Kendall's classical book [5], (1948 first edition, 1955 second edition, and 1975 fourth edition) is devoted to correlation of rank orders. Approximately half of the book is devoted to methods to calculate correlations between two rank-order sets. For the problem of the correlations between multiple rank-order sets, Kendall presents, explains, and proves the coefficient of concordance, W, method; the partial rank correlation method; the number of circular triads calculation, and the coefficient of agreement, u, method.

In 1952, Ehrenberg [154] showed the relation of Spearman's correlation coefficient ρ to Kendall's coefficient of concordance, W, which is

$$\rho_{AV} = \frac{mW-1}{m-1} \quad .$$

The paper also develops further and encourages the use of the coefficient of agreement, r, which is based upon the number r_{ij} of the m judges who agree on ordering object i higher than j.

In 1956, Siegel [155] presented several types of rank-order correlation relations to determine consistency between rank orders. These include the Spearman rank correlation coefficient, τ_{si} the Kendall Rank Correlation Coefficient, τ, and the Kendall partial rank correlation coefficient, $r_{xy,z}$, all of which are limited to analysis of only two

rank-order sets. For the relations between several rank-order sets, the method presented is the Kendall coefficient of concordance, W.

In 1957, Edwards [156] included several tests for rank-order correlation. These include Kendall's Circular Triads and the coefficient of consistency and Kendall's coefficient of agreement.

In 1963, Edwards [157] in Chapter 19, described significant tests for ranked data. These methods include r_s' the rank correlations coefficient of concordance, W_c, a continuity corrected W, tie corrections, and several tests that are limited to cases where only two-rank order sets exist.

In 1971, Conover [158] presents the same three methods for measures of rank correlation between two rank-order sets: Spearman's, Rho, Kendall's Partial Correlation Coefficient. For rank correlation between multiple rank-order sets, Conover presents the Kendall coefficient of concordance, W, and the Friedman Test, T, which is related to W by

$$W = \frac{T}{b(K-1)}$$

where K is the number of alternatives and b is the number of rank-order sets.

In 1975, Lehmann and D'Abera [159] extended rank-order correlation methods for multiple rank-order sets. The methods described are the Friedman's Q statistic method (which relates closely to Kendall's coefficient of condordance), Cochran's Q* statistic, and McNemar's Q* statistic.

In 1977, Hollander and Sethurman [160] developed a new test for agreement between two sets of rank orders by adopting a procedure proposed by Wald and Wolfowitz in 1944.

C. Single Peakedness

The monotonic trend of a set of rank orders defined as "single peakedness" was identified early by Duncan Black as one of the restrictive characteristics that improve the chance of achieving a well-defined social choice.

In 1948, Black [161] attached greater importance to geometrical than to arithmetical treatments. He said geometry will reveal logical relations better than arithmetic. A single-peaked curve is, by Black, a curve that is always upsloping or always downsloping, or a curve that is to begin with upsloping and then downsloping. Thus a single-peaked curve is any curve that shows at most a single change of direction, from up to down. The geometrical theory applies when the member's preference curves are single-peaked. Otherwise, Black says an arithmetical (matrix) theory provides the answer in any particular

problem. The matrix used is a table to express the number of votes cast for each motion (alternative) when it meets any other in a vote.

This paper contains theorems and lemmas that show the relations in the geometry and the majority situations.

In 1948, Black [9] expanded his single-peaked analogy [161]. He made the claim that when members' preference curves are single-peaked, it can be shown that voting between the different motions obey the transitive property and that if (of any three values a_1, a_2, a_3) a_1 can defeat a_2 in a vote and a_2 can defeat a_3, then a_1 can defeat a_3.

This theory applies to a decision taken on any topic by means of voting, as far as the assumptions are realistic.

In 1971, Dummet and Farquharson [13] presented one analysis of single-peakedness as defined by Black and Arrow. Dummett and Farquherson reported that Black's condition was that it should be possible to arrange the outcomes along the X-axis in such an order that every preference curve has a single peak. Arrow's single-peaked condition is slightly weaker than Black's in that it allows a peak to consist either of a single point or of two adjacent points (between which the voter is indifferent). Dummett and Farquherson continued in their paper on single-peakedness concepts to develop conditions for stability in voting. A situation is said to be unstable if there is some group of voters who, by voting differently, could have obtained an outcome they all preferred, on the assumption that all of the other voters would not change their votes.

In 1972, Kirkwood [162] provided an example of single-peakedness, which required that there be a scale along which possible alternatives a_1, a_2, "a_n" may be arranged, but not necessarily in order, such that a graph of relative preferences for the various alternatives for each judge in the group had a single peak. The same arrangement of states along the scale must be used for every judge in the group although the peaks may be in different places for different individuals.

In 1975, Denzau and Parks [12] presented a series of theorems and corollaries regarding characteristics of single-peaked rank-orders. If D is a K-tuple of weak preference relations, then (X,D) is single peaked IFF if there is a linear order $\leq$ on X such that for each $i \in K$ there are unique a_i, b_i, $\in X$ with $a_i \leq b_i$ such that

(a) $x < y \leq a_i$ implies yP_ix ;

(b) $a_i \leq x \leq y \leq b_i$ implies xI_iy ;

(c) $b_i \leq y < x$ implies yP_ix, and

(d) $(x < a_i \text{ or } b_i < x)$ and $(a_i \leq y \leq b_i)$ implies xP_iy or yP_ix.

In 1974, Chamberlin [163] summarized the single-peaked literature, in his dissertation, as it affects social choice equilibrium, by which is meant a social state from which no majority favors some "small" change. He concluded from Black and Arrow's work that, for a single dimensional issue space with single-peaked preferences, a well-defined social choice exists, and that this state is the median preference peak if the number of individuals (judges) is odd.

In 1976, Blin and Satterthwaite [11] disagreed with previous work of Dummett and Farquharson and later Pattanaik in that Blin and Satterthwaite stated that single-peakedness of preferences alone, without a restriction on admissible ballots, was insufficient to guarantee strategy-proofness. They showed this by theoretical formulation of voting procedure concepts then construction of examples where an individual had an incentive to manipulate through misrevelation, despite the existence of a single-peaked preference profile.

The procedure they used is one of the few clearly written majority-rule models in the literature. Their majority rule with Borda completion calculates the group's choice in the general steps that follow:

a) Assume a ballot profile.

b) If a Condorcet winner exists, then that is the group's choice. A Condorcet winner is that alternative which defeats every other alternative on the basis of simple (binary) majority rule.

c) If a voting paradox occurs, then there is no Condorcet winner. Proceed to a Borda count as a secondary rule.

d) The Borda count selects a winner by assigning points. The winner has the most points.

e) If two alternatives receive the same number of points, then the ballot of the group chairman (dictator) is used to break the tie. If both the admissable preferences and admissible ballots are restricted to be single-peaked, then majority-rule with Borda completion is strategy-proof.

Except for Step e), the Blin and Satterthwaite method is the same as Black's recommended majority rule method [8].

D. Majority Rule Methods

As a result of the fundamental social choice theory, several majority-rule models have been developed. These were reported in the papers which follow.

In 1948, Schuyler [14] proposed a method for rank ordering sets of research projects by aggregation of the judgments of several judges. The judges were permitted to group ties and omitted undecided alternatives. Individual rank-ordered votes are assigned sequential numbers, adjusted by a median adjusted equation, then aggregated by summing the vote numbers for each project. By this method, project individual ranks numbers are considered to differ by unit values.

In 1967, Harnett [15] developed a method called a level of aspiration model which he overlays on a majority-rule type aggregation of preferences. The method is based on responses to choices between various alternatives after they have been rank-ordered. For example, after ranking three alternatives, a judge then decides whether he would prefer his second choice for sure, which would indicate that the largest distance in utility is between Choices 2 and 3 and that his second alternative is his "aspiration level." This provides a relative utility for his choice and gives another degree of refinement in the aggregation process. Where three alternatives and three judges can give 26 rank-order arrays, including indifference, the added aspiration level parameter can give 56 different preference arrays.

In 1968, Shannon [16] presented a method to determine the level of agreement of individual rank orders and to determine the aggregated group rank order. Kendall's coefficient of concordance method is used to calculate a level of agreement. An adoption of Arrow's majority-rule method is used to aggregate the individual rank orders. First an array is set up to count the number of times each entity was preferred to the others. Next a new array is set up where a 1 replaces the dominating number in a complementary pair of alternatives relations (i.e., aPb is complimentary to bPa). A zero replaces the dominated number in the complementary pair. The row values of the new array are summed for each alternative. The relative values of the row sums determine the aggregated rank order. This order gives preferred status to the alternative which dominates the largest number of alternatives.

In 1969, Taylor [164] wrote this paper to offer a proof of the results of a 1969 paper on majority rule by Douglas Roe in the American Political Science Review, Vol. 63. The proof documented by Taylor is that when the decision rule is majority-rule, then the individual will have the lowest probability of either supporting an alternative which the group rejects or opposing a proposal which the group imposes.

In 1971, Taylor [165] thoroughly reviewed the mathematics of politics. He developed the social choice impossibility theorem concept of Arrow, the three valued (+ 1, 0, - 1) majority-rule, majority-rule equilibrium rules, special models, strategic voting, game theory, and concluded with a discussion of the application of aggregate public choice concepts to the arms race and arms control.

In 1972, Bartoszynski [166] studied the theories of dichotomous decisions, "yes" or "no." A consistent set of theory is established and related to majority-rule theory.

In 1973, Hamada [167] studied the extent that the simple majority decision rule is workable to aggregate various typed of inequality aversions among individuals. Even with very strict assumptions on the preference ordering on income distribution, voting fails for sophisticated consideration of income distribution. Voting succeeds to construct a preference ordering only for very limited ranges. If the number of income brackets (alternatives) is four or less, simple majority gives transitive ordering for an odd number of voters.

In 1973, Smith [17] developed a preference aggregation procedure and theory based on a point system. Runoff systems of aggregation are also discussed. A point system, as described by Smith, is a procedure in which a candidate (alternative) is assigned a certain real number of points for each first place vote he gets, a lesser number of points for each second place vote, etc. The points are totaled and the candidates are ordered according to the total number of each received. The candidates with the most points is first. A plurality electorial system, in which candidates are ranked according to the total number of first place votes is a point system with one point for first place and zero for anything else.

Smith presents a point runoff system where group decisions are made by successively eliminating some of the possibilities, such as for a series of elections in which low ranking candidates in one election are not on the ballot in the next election. By a similar process, one can return the discarded candidates in their individual orders to determine second place, etc.

In 1973, Cook and Saipe [168] developed the concept that the median ranking of a matrix presentation is an aggregation of individual rank orders. Theoretical results with median ranks are given as well as a branch and bound algorithm for computing a median ranking.

In 1974, Fine and Fine [127] investigated social position rules to produce an output for any configuration of individual preference orderings. The positional aspect derives because the output produced depends only upon the positions occupied by each alternative in the individual preference orderings. By the positional rule, one alternative is as good as another, if any individual's ranking of a second alternative can be matched by as high a ranking of the first alternative by some different individual.

In 1975, Fishburn [169] studied and developed a theory for a representative voting system which was interpreted as a hierarchial system in which outcomes of a vote in lower levels act as votes in higher levels of the system.

In 1975, Blin and Winston [170] applied the concepts of the discriminatory power of a pattern classifier function to majority voting decisions. A family of discriminant functions was proposed to resolve intransitive social ordering cases. This paper analyzed the problem

of preference aggregation by pattern recognition methods and leads to a derivation of a more refined class of discriminant functions which are sensitive enough to break up circular group preferences and find unequivocal classification for the most preferred group ordering.

A complete preference ordering may be represented as $\binom{M}{2}$ dimensional binary Boolean vectors (pp. 559) which are preference patterns. Such patterns are characterized by features which correspond to an elementary preference between two alternatives. Through this machanism, the majority principle can be stated as a simplified patterns classifier. The output is a binary vector that corresponds to either a unique ordering or a finite union of a class of preference orderings.

In 1975, Straffin [171] studied dichotomous decisions by an n-member body. For an odd number of voters, it was proven that majority rule maximizes the average responsiveness of the body to individual preferences. Other conditions cause majority-rule vote not to be a unique decision rule when n is even.

In 1977, Sheridan and Sicherman [172] described a method for electronic real time, anonymous voting, estimation of a group decision. The group was regarded as an entity whose preference response was, constrained to a single scalar measure of the degree to which it agreed with either one of two choices. They related the measure of votes percentage to the membership function of fuzzy set theory.

In 1977, Armstrong and Cook [18] reported on investigations of committee ranking where a given member wishes to select a ranking of a set of alternatives which, when combined with the other member's rank orders, yields a transitive consensus ranking that is as close as possible to some desired set of preferences. The method utilizes a branch and bound algorithm. A comprehensive example is given. In addition the paper presents two weighting approaches:

1) An exponential weighting of one member's rank order over the others.

2) A multiplicative weight to consider the relative importance of pairwise preferences.

Wood and Wilson [19] reported on methods to convert rank orders into measures. They can be used where ranked data must be amalgamated with interval data. The methods require ranked differences between alternatives. Three methods are compared in the paper, using empirical data and a known cardinal rank as a reference. The first method is one by Kendall where judges rank not only alternatives but also rank differences between alternatives, and even rank the differences of differences.

In the second method, the rank swap method, the highest scale value is transferred to the alternative ranked first, the second highest value is transferred to the second ranked alternative, etc. The third method, the normal scores method, replaces ranks by order statistics from a known distribution such as the normal. The Wood and Wilson study demonstrated that the Kendall method agrees best with the reference data.

In 1978, Campbell [173] developed the theory for computationally viable choice functions which did not waste time generate reasonably satisfactory intermediate alternatives, and adjusted easily as new alternatives became available or as old alternatives were deleted. Several procedures were considered, but the binary procedure was analyzed most as the preferred. The required features for computation were defined as a basis for theorems to prevent irrational functions. An example was not given.

In 1978, Grandmont [174] developed possibility theorems for majority rule based on the shape of the distribution of individuals. Single peaked preferences are a particular case of the theory developed.

In 1979, Navarrete, et al. [175] presented another method of aggregating rank orders. Their method was based on a ranking indicator determined from the alternatives dominated and the alternatives dominating. For strong orders, the method was the same as the Shannon [16] method plus the determinations of the rank indicator. The indicator for this case is the difference of the PREF matrix row sum for an alternative versus the column sum for the same alternative. For weak orders, the matrix scoring differs from the Shannon method. Navarrete presented variants of the method which become majority rank methods when developed. Although clearly explained, little justification was made for the chosen and variant methods or any other method.

E. Number of Voters and Alternatives

Several papers analyze situations where the number of voters or the number of alternatives reach a level where aggregation characteristics change their trend. These papers follow.

In 1948, Black [10] developed the theory and logic to determine the elasticity of committee decisions with the majority-rule technique. The ratio to measure the elasticity of the committee's decisions with an altering size of majority can be represented as a coefficient. This coefficient is the ratio of the percentage of increase in the maximum size of the variable, which remains immune from changes, to the percentage of increase in the size of the majority needed to alter it. With this m/n ratio, Black developed the desired elasticities for economic analysis.

In 1973, Fishburn [20] examined the effects of the degree of homogeneity among voters' preference orders on the probability that some alternative will defeat every other alternative on the basis of simple majority, and the degree of common winners from the Borda and Copeland methods of social choice. In all cases, the number of voters was odd and had linear preference order. Three measures of social homogeneity were compared: the Kendall-Smith coefficient of concordance (W) for a given profile; the maximum number of voter preference orders that were jointly single peaked for a given profile; and a Jamison and Luce parameter ($1/\sigma$) which indicated the degree to which voters' preference orders may be similar. In each case the tendencies were consistent and the probability for a simple majority winner increased as the index of homogeneity increased.

In comparison of the Borda and Copeland methods, the propensity for complete agreement between the two methods appeared to increase as W increased. There was some agreement tendency when there was a high W or a very low W, denoting disagreement. Under Copeland's method, the winners were the alternatives with the greatest number of simple majority wins over the other alternatives. A simple majority winner will be a Copeland winner, but may not be a Borda winner. For $n < 7$, $m < 5$, 97% of the cases have the same Borda and Copeland winner; for $n < 21$, $m < 9$, the average is 84% like winners.

In 1974, Fishburn [21] investigated multi-dimensional single-peaked preferences which determined the probabilities of cyclical majorities. The results showed that, if the number of voters was much larger than the number of alternatives in the two attribute cases, the probability of no simple majority winner will be exceedingly small. Also, if the number of voters and alternatives were roughly equal, the no-winner probability increased but generally remained smaller than the no-winner probability in the impartial situation. If there were three alternatives and preferences were single-peaked in each of two attributes, then a cyclical majority might arise within a small committee; however, the probability of getting a cycle was less than the probability of a cycle with the impartial culture.

In 1976, Fishburn and Gehrlein [22] developed the two-stage probabilities involved in a two-alternative election in which voters were uncertain about which alternative (candidate) they will vote for and the winner was to be determined by a lottery between the alternatives that was based on their vote total. The derivations were based upon μ (n) which was the smallest number for which the simple majority rule maximized one alternative's win probability.

In 1978, Bell [176] extended theory to investigate the number of alternatives, in the winning cycle in a pairwise vote sequence, when there was no Condorcet winner. The Condorcet winner had a simple majority against any other alternative. He discovered that the number

of alternatives in the top cycle was always likely to be n - 1 or n (where n is the number of alternative) than some number between 3 and n - 2 inclusive. Thus most of the time when majority rule broke down, either n - 1 or n alternatives were involved in the cycle that included all potential winners.

Bell presented, in tabular form, the probabilities of K possible winners out of n alternatives for n from 3 to 80 and for K from 1 to 80.

In 1978, Dutta and Pattanaik [24] critiqued earlier papers concerning "nice" or Nash equilibria. They established that a Nash equilibrium can be found for every non-dictatorial procedure and for every sincere preference profile. Also, they showed that for a large number of alternatives, for some sincere preference profiles, no equilibrium (winner) may exist under many decision procedures.

In 1979, Greenberg [25] proved that a d-majority equilibrium existed when the domain was a compact and convex set of dimension m. A d-majority equilibrium was an alternative X such that no other alternative strictly preferred X by at least d voting individuals. The d-majority equilibrium can exist when d is greater than $[m/(m + 1)]n$.

When the preferences were not strict, that is, they had indifference or were incomplete, Greenberg developed δ relations to substitute for d. A δ-relative equilibrium can exist whenever $\delta \geq \min [n - 1, m]$.

F. Arrow's Impossibility Theorem

Kenneth Arrow's Impossibility Theorem [56] impacted the aggregate social choice theoreticians so significantly that many interpretations, clarifications, and critiques were written in subsequent years. Arrow's theoretical writing style stimulated many of the follow-on articles. The turmoil Arrow's book and Theorem caused in economic and political scientific practice prompted intense efforts to destroy or dilute Arrow's thesis. Minor clarifications were presented and accepted by Arrow in his 1963 second edition [26]. Arrow's works have survived the intensive attention and still stand as theoretical classics in the field. Several of the articles which comment on Arrow's Theorem are discussed in the following paragraphs.

Arrow's second edition of his classical book Social Choice and Individual Values [56] set forth the theory of establishing the consistency of a group by collective modes of choice by the wills of many people. Arrow defined the conditions necessary for a social welfare function. They are as follows:

1) Every logically possible set of individual orderings of three alternatives can be obtained from some admissible set of individual orderings of all alternatives.

2) If one alternative social state rises or remains still in the ordering of every individual, without any other change in the orderings, then it rises, or at least, does not fall, in the social ordering.

3) Generally, all methods of social choice are of the type of voting, thus giving independence of irrelevant alternatives.

4) The social welfare function is not imposed.

5) The social welfare function is not dictatorial.

Arrow then developed his General Possibility Theorem which was interpreted as:

> "If we exclude the possibility of interpersonal comparisons of utility, then the only methods of passing from individual tasks to social preferences which will be satisfactory and which will be defined for a wide range of sets of individual orderings are either imposed or dictatorial."

In 1952, Little [28] analyzed Arrow's conclusions that for aggregation of ordinal individual preferences into group social preferences, only dictatorial methods are valid. He cited Arrow's conditions:

1) An alternative's relative position in an individual order will not relatively change in the aggregate order.

2) Independence of irrelevant alternative.

3) Nonimposition.

4) Nondictatorship.

Arrow proved that no consistent collective order satisfies all of these conditions. Little states that Condition 1 is inapplicable if tastes are given; therefore, Arrow's results do not bear on Bergson's social welfare functions.

Little also concluded that Arrow's system cannot be interpreted as a critique of social welfare functions and it is applicable to decisions-making processes as minimum conditions for a satisfactory democratic decision-making procedure.

In 1954, Buchanan [29] examined Arrow's arguments to reveal a weakness in Arrow's formal analysis. He said that some implications that have been drawn are inappropriate. Buchanan further concluded that the market is not "collective choice." Further, he said that Arrow's arguments have been erroneously interpreted as proving that

the preference decision-making processes are irrational. Buchanan said Arrow's work proved that aggregated individual orders allow decision-making processes but not of the type produced by utility measures.

On majority decision choice, Buchanan says it is limited by its accidental nature, as caused by the paradox; majority rule is acceptable because it is subject to revisal and change.

In 1959, Arrow [27] wrote that definitions can be expanded for "demand functions" and "orderings" which had been theoretically analyzed separately. Arrow contended and proved that the demand function theory would be simplified if it was considered to include all finite sets. His conclusion was that the Weak Axiom of Revealed Preference was completely equivalent to the existence of an ordering from which choice functions can be derived. He finally observed that the use of experimental methods for preference studies will require inferring from finite choice sets on to infinite choice sets.

In 1961, Murakami [32] presented some logical property extensions of Arrow's social welfare function for economists. Arrow's function can also apply to political and sociological analysis. Murakami proved that Arrowian social functions can be expressed as a system of two-valued logical functions, such as the two values of X being "R" or "not R." He showed that any two-valued logical functions were expressible by majority decision operation and negation operation. He concluded that Arrowian social welfare functions had a kind of widely defined majority decision structure. Murakami further demonstrated the self-dual and neutrality characteristics of Arrowian majority decision logic.

In 1969, Hansson [46] gave another proof of Arrow's well-known results. Arrow was defended where it was claimed that his theorem was irrelevant because it depended on intransitivities among the alternatives, which in turn depended on the effort to obtain an ordering. Arrow was also defended where his theorem was claimed as irrelevant because it presupposed the same rationality concept for the society as well as for the individual.

In 1972, Davis, De Groat, and Hinich [31] formulated a fresh look at the problem of social choice. They stated and proved necessary and sufficient conditions, such as dominance, symmetry, and unique median which had a transitive social preference ordering.

In 1972, Schwartz [36] analyzed decision-making principles (DMP) to determine their underlying similarities such as noncircularity, nondominance, and rationality. Rationality has three conditions: weak dominance, narrowness, and reducibility. Schwartz also stated the axioms for a set of optimal elements with respect to preference.

In 1975, Ostojic [30] in his paper on information for decision making in the Yugoslavian economy reflected on Arrow's theorems on the aggregation of conflicting preferences. He considered Arrow's assumptions to be very restrictive and unrealistic. The major conflict was Arrow's position that a decision requires a "benevolent dictator" or an economic agent to determine social choice rules. Ostojic contended that the economic agents make fundamental decisions themselves and compare their mutual preferences via bargaining. This was obtained through goals which would link local and global interests.

In 1975, Farris and Sage [33] surveyed group decision making using Arrow's concepts and theorems as a basis. For Arrow's Social Welfare Functions conditions, his Impossibility Theorem was accepted. But for relaxations of Arrow's Condition 3, first, or Condition 1, social welfare functions could be developed. The paper also thoroughly reviewed Black's single-peakedness and odd-even number of voters theories.

In 1976, Keeney [62] developed the conditions, parallel to Arrow's, for social preference aggregation where the individuals had cardinal utilities rather than ordinal rankings. For both certain alternative models and uncertain alternative models, Keeney showed that it was always possible to define consistent aggregation rules for a group cardinal utility function.

In 1977, Ferejohn [34] analyzed the work of D. J. Brown [104] and B. Hansson [114] in light of Arrow's conditions for social preference aggregation. Ferejohn concluded the Brown's approximate solutions to Arrow's problems were essentially dictatorial. Further, he concluded that the maximal socially decisive nondictatorial procedures are almost the same as the dictatorial procedures.

In 1978, McGuire and Thompson [35] characterized an evolutionary order from various taxonomic criteria by applying the concept of Arrow's Impossibility Theorem. The analysis was in terms of multiple species, S, S', S", etc., and taxonomic criterion, T. Conditions paralleling Arrow's conditions were defined. Then an impossibility theorem was developed to show that the conditions were inconsistent.

G. Extensions and Revisions of Arrow's Impossibility Theorem

In addition to stimulating clarification or critique articles, Arrow's Impossibility Theorem also served as the basis for many articles which recommended revisions to the theory. Many of these developed modified conditions for social welfare functions that would permit a Possibility Theorem to be proved. Other articles strived to extend the Impossibility Theorem to include new or revised conditions based on additional characteristics of individual and group preference orderings. Many of these articles are summarized in the following paragraphs.

In 1952, May [37] defined the conditions for simple majority rule and analyzed Arrow's theories further. The required conditions for a simple majority group decision function are decisiveness, symmetry, neutrality, and positive responsiveness.

In 1952, Goodman and Markowitz [38] developed three resolutions which, when integrated into Arrow's conditions, created a revised set of conditions that could be satisfied by social welfare functions. These resolutions influence Arrow's first three conditions most. In addition, this paper defined and used, as an example, a potential social welfare function, called "Summation of Ranks." This Summation of Ranks method contradicts Arrow's Condition 3, while the method of majority rule contradicts the transitivity property of Condition 1.

In 1953, Hildreth [47] developed a set of conditions that can be satisfied by von Neumann-Morgenstern utilities. These conditions conflict with Arrow's Condition 3. Hildreth compared his conditions with those of Arrow and presented rationale for the differences.

In 1957, Blau [39], in another classical social choice paper, analyzed and pointed out limitations in Arrow's theorem. Blau first paraphrased Arrow's conditions as: "1. A triple is free if all conceivable combinations of individual orderings of this triple actually occur; 2. Quasi-monotonicity; 3. Binary choice; 4. Citizen's sovereignity; and 5. Nondictatorship." Blau developed a unanimity rule as a consequence of Conditions 2 and 4. This rule is called a "Unanimity Rule for Preference (URP)." He further extended Condition 2 to 2' monotonicity. Blau then proved that an impossibility theorem for a social welfare function must be based upon satisfying Conditions 2', 3, 5, and URP. In this paper, Blau further established definitions for neutrality, dictatorial, oppressed, indecisive, and additive.

In 1964, Inada [48] broadly analyzed economic welfare functions. A portion of this paper is devoted to a discussion of Arrow's and Blau's social welfare function conditions. Inada generally agreed that Arrow's conditions required changes more like Blau's conditions.

In 1964, Inada [40] extended Arrow's SWF theorems for the Simple Majority Decision Rule. By classifying all alternatives into two groups where all were indifferent among each group, the simple majority decision rule was a social welfare function which satisfied Arrow's Conditions 2 through 5. If any three alternatives satisfied single-peakedness and the number of individuals was odd, then the simple majority decision rule was a social welfare function satisfying only Arrow's Conditions 2 through 5. Inada, in this paper, also used single-caved preferences, an opposite concept to single-peaked preferences, which was theoretically applied to simple majority decision rule SWF theorems in place of single-peakedness.

In 1966, Sen [177] extended the work of Arrow, Black, and Inada on the conditions for a majority decision preference to be an SWF. The value-restricted preferences condition was defined to require a triple to be either best, worst, or medium value in an individual preference ordering. Sen presented his Possibility Theorem for Value-Restricted Preferences which stated the conditions for a majority decision to be a social welfare function. Sen claimed that his theorem covered the conditions of Arrow, Black's single-peakedness, Inada single-caved preferences and separable group conditions, and Ward's Latin-squarelessness.

In 1969, Hansson [46] questioned Arrow's third condition (A3 independence of irrelevant alternatives) and demonstrated conditions, NA and NP, where alternatives and persons were equal. He then proved that "no democratic voting function can fulfill Condition A3 unless NA and NP were overcome by assignment of the resulting preference ordering.

In 1970, Sen [178] thoroughly analyzed the theory caused by varying degrees of interpersonal comparability of the welfare measures of individuals.

In 1970, Fishburn [45] developed an alternate proof of Arrow's Impossibility Theorem. The alternate proof began with the nondictatorial condition and showed that the first "voters" condition was incompatible unless the number of voters was infinite which is a contradiction.

In 1972, Wilson [49] proved Arrow's General Possibility Theorem without assuming the "Pareto Principle" as Arrow had used. The "Pareto Principle" asserted that society preferred alternative unanimously preferred by its members. Also "Citizen's Sovereignty" need not be assumed.

In 1972, Dalkey [179] extended Arrow's Impossibility Theorem to incompass group Delphi probability aggregations.

In 1973, Campbell [41] attacked Arrow's third condition, "A_3," which was the "independence of irrelevant alternatives which prevented the collective-choice rule from responding to the intensity of individual preference." Campbell developed a modified version of A_3, which he named A*, which took intensity into account but, he claimed, still provided the original independence condition for individual preferences and the rankings were still ordinal.

In 1974, Kuga and Nagatani [42] developed a "possibility theorem" to counter Arrow's Impossibility Theorem on group social welfare functions. The approach by Kuga and Nagatani was to define the index of the degree of intensity of antagonism between individual opinions. This index of antagonism was inversely related to the similarity of individual

preferences. The possibility of a social welfare function was defined as the complement of the "paradox gauge" which was the percentage of individual preference group patterns that lead to the voting paradox. Next, the authors proved that the paradox gauge varies directly with the antagonism index; i.e., there was a greater possibility of voter's paradox when there was more antagonism between individual voters. Finally, the authors determined a sufficient condition theorem in terms of the antagonism index for non-occurrence of the voting paradox. For three alternatives for an odd number of voters, the paradox could occur when the antagonism index was greater than $2/3(N^2/N^2 - 1)$. For an even number of voters the paradox could occur when the antagonism index was greater than 2/3. The antagonism index was determined by a combination of the count of the number of individuals whose preference was of each possible type of strong order for the set of alternatives.

In 1974, Ferejohn and Grether [43] developed alternate conditions to Arrow's conditions for the possibility of obtaining a group social welfare function, SPR. Their conditions were anonymity (A), neutrality (N), and positive responsiveness (PR). The authors considered their method equivalent to weakening of the methods of May and Sen.

In 1974, Feldman [44] provided a proof of Arrow's Theorem with excellent examples. The proof examined the five conditions for a social welfare function: 1) Completeness and Transitivity, 2) Universality, 3) Pareto Consistency, 4) Nondictatorship, and 5) Independence of Irrelevant Alternatives. The logic lead, in all cases, to making one individual a dictator. Further, the author showed how the "Universality" requirement could be weakened without weakening the theorem.

In 1974, Salles [109], in a note added one preference to a 1970 paper by Inada. This note demonstrated that quasitransitivity was violated.

In 1975, Bacharach [180] studied various axioms of group choice to determine which combinations of axioms lead to preferred consensus methodology. The conditions that Bacharach considered are Group Rationality (GR), Unanimity of Probabilities ($UNAN_\pi$), Unanimity of Utilities ($UNAN_u$), No Dictator of Opinions ($DICT_\pi$), Independence of Irrelevant Alternatives (IIA), Strong Pareto Criterion (PAR_s), Weak Pareto Criterion (PAR_w), Unanimity of Indifference (PAR_n), Column Linearity (LIN), Continuity (CON), and Equivalence of Identical Assessments (EQUIV). The paper then presented several theorems which combined certain of the preceding conditions to establish a restricted group social welfare function, i.e., Theorem 3 said that if the individual orders obeyed PAR_n, GR, $UNAN_u$, IIA, and EQUIV,

then $\pi^G = \sum_1^P \lambda_p \pi^P$ where λ is fixed ($\lambda = 0$, $\Sigma_1^P \lambda_p = 1$) where π is a probability measure, P means personal, and G means group.

In 1975, Wilson [181] extended Arrow's Impossibility Theorem for aggregation of individual preferences into a group preference ordering to a general theorem for aggregation in an arbitrary domain. This was done through the determination of the algebra which was included in Arrow's Theorem.

In 1976, Parks [182] showed that a social welfare function was "dictatorial if it has an unlimited domain, is independent of irrelevant alternatives, Pareto Consistent, and there are at least three alternatives." Also, Parks showed neutrality to learn inherent conditions resulting from the specified conditions. Second, Parks showed that the Bergson-Samuelson welfare function was also dictatorial, based on results studied for a fixed m-tuple.

In 1976, Blair, Bordes, Kelly, and Suzumura [183] studied several methods of weakening the collective rationality condition in Arrow's Impossibility Theorem. The conditions held constant were unrestricted domain, nondictatorship, the Pareto condition, and independence of irrelevant alternatives. The collective rationality condition was studied in its two parts: rationality and the transitivity and connectedness of the rationalization. Approaches tried for relaxation were Plott's "Path Independence," the Chirnoff condition, and the Generalized Condorcet Property. With all of the relaxations considered, the Impossibility Theorem still held.

In 1978, Monjardet [184] published a French article which contained a new proof of Arrow's Impossibility Theorem. The proof is based on two independent lemmas: one based on an antireflexive binary relative and another based on an "ultra-filter."

In 1978, Blin and Satterthwaite [185] developed new derivations of Arrow's Impossibility Theorem for strategy-proofness in group decisions. Their purpose was to show that groups "cannot make decisions in the same rational, straight-forward manner that an individual can. They show "that a correspondence exists between the properties that group choices would exhibit if they did satisfy the rationality and independence of irrelevant alternative conditions that Arrow postulated."

H. General Social Choice Theory Overviews

Several books and articles broadly covered the general theory of the aggregation of individual preferences into a single group preference order. A few emphasize a specific area with a broad background coverage. These are highlighted in the following paragraphs.

In 1953, Von Newmann and Morgenstern [186] developed the basic game theories which are used in one branch of individual preference analysis.

In 1957, Luce and Raiffa [50], in their Chapter 14, developed the basic concepts of Group Decision Making. They began with Arrow's formulation of the social choice problem and the Impossibility Theorem. Then after describing cardinal techniques, the chapter described the majority-rule model and the game model.

In 1964, Aumann's chapter in Shelly and Bryan's book [187] briefly described ordinal utility ranking problems then proceeded to describe an additive structure where utilities could be added.

In 1971, Pattanaik [51] developed comprehensive formal theories of voting and collective choice. This included a thorough analysis of Arrow's Impossibility Theorem, restrictions on rank orders to achieve usable aggregations, Inada's work on majority rule, single-peakedness, other conditions for aggregation, and individual intransitivity.

In 1973, Fishburn [53] presented a comprehensive theory, with examples, for social aggregate choice. The book was in three major parts: social choice between two alternatives where he covered duality, coalitions, weighted voting, simple, weak, and strong majorities; simple majority social choice where he covered binary relations, single-peakedness, transitive majorities, odd-even number of voters, Condorcet conditions, and other historical works such as Borda and Dodgson; and social choice functions which included conditions for transitive, equilibrium results, and Arrow's Impossibility Theorem. This book is an excellent balance covering all significant social choice theory to date with inclusion of adequate examples to make a practical base.

In 1973, Herzberger [54] in a most thorough theoretical paper developed the definitions, propositions, and theorems to test the concept of ordinal rational preference choice.

In 1973, Blin, Moberg, Fu, and Whinston [188] developed a model for collective social choice. The model used the individual binary matrix T, with $t_{ij} + t_{ji} = 1$, based on comparisons and preference choices of each pair of alternatives by each judge. The binary process then lead to a linear aggregation matrix rule over the individual T's. The aggregate order was determined by the optimal completion of the aggregation matrix. This paper incorporated an optimization concept to achieve the optimal. Then three significant theorems were developed:

1) A transitive aggregate ordering is an optimal solution.

2) With single-peakedness, majority rule will always achieve an optimal solution.

3) If multiple optimal solutions are achieved, then majority rule will give an intransitive aggregate rank order.

In 1978, Pattanaik [52] extended social choice theory to include game theory for group decisions; group decision functions such as majority rule, non-minority, pairwise characteristics, sequence voting, and restrictions for a choice; and strategic manipulations and proofness, and Arrow's Theorem.

In 1978, Ferejohn and Page [55] developed this paper to discuss the changes in social choice functions over time steps. In preparation for this time-related analysis, they established a broad base of aggregate social choice theory. This included binary pair comparisons, Chakravarty's dominance rule, Arrow's social welfare function conditions and his Impossibility Theorem, and Koopman's discounting rule for preference analysis.

I. Social Welfare Function

In 1951, Arrow [26] published extensive theory developing and redefining the social welfare function (SWF) which is used often in welfare economics theory to describe analytically the preferences of a society in a welfare economy. Many articles have been written to clarify, expand upon, or modify Arrow's SWF theories. Many of these articles are highlighted in the following paragraphs.

In 1953, Rothenberg [189] analyzed SWF and appraised Arrow's work on that subject. He stated that the premises from Bergson's 1938 formulation of the social welfare function had been attacked and threatened by claims of excessive abstractness and clumsiness and by Arrow's attack through his Impossibility Theorem. Rothenberg interpreted SWF further and recommended alternate concepts that would be less vulnerable to these attacks. He further defined a social welfare function, not as a tool of welfare analysis, but only as a descriptive generalization about the valuation rule of a population and that it was empirically observable.

In 1957, Blau [39] analyzed Arrow's Impossibility Theorem and demonstrated an example where the theorem was false and an SWF did exist. In addition, Blau defined a revised Arrow Condition 2' (Monotonicity) instead of Condition 2 (Quasimonotonicity) and a Unanimity Rule for Preference (URP) both of which can make Arrow's Impossibility Theorem true: "If D is universal, no SEF can satisfy Conditions 2-5" and "If D is universal, then no SWF can satisfy Conditions 2', 3, 5, and URP."

In 1970, Fishburn [58] presented a single theorem tc define the conditions of a group decision function as a representative majority group decisive function. These conditions were duality, monotonicity, unanimity, and coalitions.

In 1971, De Meyer and Plott [57] developed a voting process of social welfare functions that incorporated "intensity" of preference through matrix methods. The relative intensity a_i/a_j was used as the ratio of preference a_i over the alternate a_j. The authors established that the relative intensity matrix vectors provided a social welfare function.

In 1974, Young [61] presented a definition of proof of a subpreference function for aggregating preferences. This theoretical approach is useful in substantiating social choice function theory.

In 1976, Kemp and Ng [190] presented propositions to cast doubt on the existence of Bergson-Samuelson SWFs. They demonstrated that it was impossible to find a "reasonable" Bergson-Samuelson SWF based on individual orderings, and these SWFs were subject to impossibility results not very different from those for Arrow type SWFs. The Bergson-Samuelson SWFs were rules for mapping any one member of a class of admissible sets of individual orderings into the set of all possible social orderings, while Arrow type SWFs were rules for mapping only one given set of individual orderings. The results challenged that the Bergson-Samuelson SWF could be derived from individual ordinal utilities. SWFs can be constructed to engage in interpersonal comparisons of cardinal individual welfares.

Samuelson [63] rejected the proposition by Kemp and Ng that the Bergson-Samuelson type of SWF was impossible or depended on cardinal preference data. Kemp and Ng's axiom for neutrality and independence, Axiom 3, was rejected fully by Samuelson as unreasonable. If Axiom 3 were true, Samuelson agreed with Kemp and Ng. Samuelson further gave an example of a valid SWF. Kemp and Ng explained in the same issue why they felt their rejection of Bergson-Samuelson SWF was correct. They stated that their Axiom 3 was correct, whether reasonable or unreasonable.

In 1968, Peleg [60] studied the conditions required to have a social choice function that is exactly and strongly consistent, and anonymous. Many "democratic" rules of voting such as plurality voting are exactly consistent, but all are not strongly consistent, which means the functions are not distorted from manipulation by coalitions.

In 1979, Gevers [59] presented two more cases where social welfare functions were based on interpersonal utility comparisons. In the first case, the planner was allowed to compare utility levels interpersonally and prevented from comparing utility gains. In the second case, case one was supplemented by allowing the planner to consider as meaningful interpersonal comparisons of utility gains of several forms.

J. Comparisons of Borda and Condorcet

A fundamental tool of majority rule aggregation is the Borda rule as conceived by J. C. de Borda in 1781 in France and later developed by Duncan Black in 1958. Many papers have been written analyzing and evaluating the Borda rule. Several of these will be discussed in the paragraphs to follow. Other papers used the majority rule method of the Condorcet as their reference base.

In 1953, De Grazia [2] translated and analyzed Borda's 1770 paper in which he developed his majority voting rule using numbered, ranked choices. But De Grazia presents several weaknesses of Borda's proposals. Namely, Borda did not define his majority principle. Borda considered adjacent ratings to be equidistant and separated, and Borda neglected the "Don't waste your vote" philosophy which might make an individual vote for other than his first choice. In summary, he felt Borda had tried to isolate the method from the individul's voting.

In 1971, Fishburn [191] presented a critical analysis of rules for aggregating rank orders by comparison of the winning alternatives. The rules were variations of the ones developed by Borda and Condorcet. For orders having permuted dominance (PD), the Borda rule produces a satisfactory answer while the Condorcet rule fails. Conversely, orders that are consistent under the reduction principle (RP) give good Condorcet, but not Borda, winners. Fishburn included the results of extensive computer simulations to determine the probability of the Condorcet rule having a winner and the probabilities that the Borda and Condorcet rules will give the same winners. This paper is also marked by clear descriptions of the variations of the rules and example problems to compare the rules.

In 1973, Crook [192] emphasized the concept of a "Condorcet Set" in his dissertation. A Condorcet Set was defined as a set of platforms (alternatives), no one of which is preferred by a majority to all other platforms in the set, but each of which is preferred by a majority to all platforms not in the set. The set satisfies the Condorcet criterion.

A Condorcet set can be determined by a two-person game. No precise model is given to determine a Condorcet set, one example uses three platforms (alternatives).

In 1974, Fishburn [193] developed a system to compute the efficiency of several weighting voting rules as compared to the simple majority rule. The types of rules were truncated constant rules and truncated linear rules (one of which was the Borda rule). On the basis of simulation data, the Borda rule was determined as the most efficient.

In the second one-stage voting efficiency study by Fishburn in 1974 [194], the efficiency of simple weighted, truncated majority rules were compared to a base of the Borda majority rule. Of the constant rule, the linear rule, or modified linear rule; the constant rule

(vote for k of m candidates) was most efficient when k was near m(1-1/n)/2 when compared to Borda. But the linear and modified linear truncated rules gave even higher efficiencies of picking the same winners as Borda.

In 1974, the object of Young's [66] paper was to present axioms that uniquely characterize Borda's rule and defined Condorcet's rule. The Condorcet function chooses those alternatives that tie or beat every other alternative under pairwise simple majority voting. The Condorcet function is not defined everywhere. For the Borda rule, given a profile W on m alternatives a score of m + 1 -2i is assigned to an alternative every time it is the ith most preferred by some voter. The Borda rule chooses the alternative with the highest total score for all voters. For the whole profile Borda Rule (or adjusted Borda as Black defined)

$$B_k(W) = \Sigma\ \pi_{kj}(W) - \pi_{jk}(W)$$

$$1 \leq j \leq m$$

$$j \neq k$$

The principle result of the paper was a Borda rule Theorem 1: for any fixed number m of alternatives, there is one and only one social choice function that is neutral, consistent, faithful, and has the cancellation property-namely, the Borda rule. Lemma 1 also stated; If f is consistent and has the cancellation property, then f is based on pairwise comparisons. Both the Borda and Condorcet rules are based on pairwise comparisons.

In 1974, Bowman and Colantoni [195] used the theorem of the alternative to show equivalence between transitive majority rule and the solution of a certain set of linear inequalities. The system of linear inequalities is with characteristics for every triple that are required for transitivity of majority rule. This permits the use of restrictions on vote distributions over the preference patterns to assure transitivity.

In 1975, Paris [72] determined the probability of Condorcet, majority, and pluratlity decisions. The study contained probabilities of equally likely preference orderings and the probability of single-peaked preferences. For example, at approximately 50% of the vote for an alternative, the Condorcet first-place alternative, as compared to the plurality winner, has the highest probability of single-peakedness.

In 1975, Young [67] developed axiomatic definitions for social scoring functions, including such characteristics as an anonymous social choice function, a neutral function, and a symmetric function. He demonstrated that for three or more alternatives there was no completely natural extension of the simple majority rule. As a result, a great variety of rules is used in practice for group decision making. This

includes the following: plurality, Borda, Condorcet, sequential voting, exhaustive voting, and the double election.

Young's main theorem is that (1) a social choice function is symmetric and consistent if it is a scoring function, and (2) a social choice function is symmetric, consistent, and continuous if it is a simple scoring function.

In 1975, Richelson [68] made a comparison of several social choice methods namely: (1) simple plurality, (2) Borda, (3) Copeland, (4) Dodgson, and (5) Black's Condorcet/Borda procedure. The methods were evaluated for several condiditons: E, an alternative, has a simple majority over every other alternative; RP, removing a Pareto dominated alternative from the domain, does not change the social choice set; M, monotonicity; A4, If $|Y| \geq 2$ and $F(Y, D) = \{x\}$, then $X \varepsilon F\ (Y - \{y\}, D)$ for some $Y \neq X$; A5, If $|Y| \geq 2$ and $x \varepsilon F\ (Y, D)$ then $x \varepsilon F\ (\{x, y\}, D)$ for some $y \neq x$; C, consistency; N, concellation, requires that any yx in an individual's preference ordering be balanced by any xy in some other individual's orderings; and PD, permuted dominance.

In summary, Methods 1 and 4 do quite poorly. Methods 2, 3, and 5 are superior. Because of condition E, Method 2 is less preferred rather than Methods 3 or 5. Method 3 versus 5 depends on the relative importance of Conditions M and A5.

In 1976, Fishburn and Gehrlein [75] continued Fishburn's analysis of voting systems from one stage to two stages. This study assessed the tendencies of several two-stage systems to elect the candidate which would be elected by a less practical procedure such as simple majority of the Borda sum-of-ranks method. Computer simulations compared the two-stage and one-stage methods for from three to ten candidates. The method evaluated cases for m candidates with x voted for on the first ballot, k candidates on the second ballot, and y voted for on the second ballot. This is denoted as m(x, k, y); i.e., 6(1, 2, 1). The efficiency term is similar to that used in Fishburn's one-stage studies, namely

$$E = (100)\ \frac{\text{Prob of elec by evaluated method}}{\text{Prob of elec by majority rule norm}}$$

A case is only considered if the norm has a winner.

When the norm was the simple majority method, key study conclusions were as follows:

1) Among two-stage systems of the form (1, k, 1), the most efficient system is the basic double plurality (1, 2, 1).

2) For $n \leq 6$, the best one-stage system has much lower efficiency than for a double plurality (1, 2, 1).

3) The best two-stage system always has the form (x, 2, 1) with efficiencies above 0.80.

4) The best one-stage system says vote for about half of the candidates.

When the norm is the Borda method, the key conclusions are:

1) Same as 1) in the preceding list for simple majority norm.

2) The best one-stage system efficiencies are larger than double plurality efficiencies for m = 5, 6, which are the m values used for Borda calculations.

3) Same as 3) in the preceding list, but Borda norm, efficiencies are in the 0.70's.

4) Same as 4) in the preceding list. This Fishburn study gave meaningful, quantitative answers.

In 1976, Moon, [76] derived the equations to determine the minimum size of a committee to achieve simultaneously any two rankings by two consistent procedures. If m is the number of candidates, when considering two methods: the sum of score (or Borda) method or when considering the sum of votes method. If the results are untied, the number of committee members to get two different rankings is m + 1 for $m \geq 2$. When ties are present, the corresponding value is m + 2.

In 1976, Gehrlein and Fishburn [121] developed the analysis models to determine the probabilities of Condorcet's paradox when there were preference profiles and Anonymous profiles. Condorcet's paradox occurred in a voting situation with n voters and m candidates (or alternatives) if for every alternative there is a second alternative which more voters prefer to the first alternative than conversely. A profile is a function that assigns a preference order on the alternatives to each voter. An anonymous profile (A-profile) is a function that assigns a nonnegative number of voters to each potential preference order on the alternatives such that the sum of the assigned integers equals m.

In 1976, in their survey of Borda's rule and Condorcet's Principle, Fishburn and Gehrlein [65] defined the positional scoring rules and the Condorcet methods and then compared their characteristics. The Borda rule and simple majority were considered to be positional rules. Four Condorcet methods are as follows:

1) Borda's Elimination Rule where a Borda loser is eliminated in each stage.

2) Black's method picks a set of candidates that are not beaten by another candidate on the basis of simple majority. If none are found, the set of Borda winners is used.

3) Copeland's method where the winner is the candidate with the maximum value of the number of candidates it defeats by simple

majority minus the number of candidates that defeat it by simple majority.

4) Dodgson's Method where the winner is the candidate with the minimum number of inversions in linear orders necessary to make it the Condorcet winner.

The study concluded; concerning "W" rules which are strictly monotonic positional scoring rules:

1) Borda's rule is the only W rule that invariably yields all candidates as tied winners whenever the number of voters who prefer one candidate to a second candidate equals the number who prefer the second to the first.

2) The Borda rule is the only "W" rule that yields x as a winner under one profile when x is a winner under a second profile and the number of voters who prefer candidate a to b is the same in both profiles.

3) The Borda rule is the only "W" rule that guarantees that a Condorcet winner will not be beaten by every other candidate under rule W.

4) A sequential elimination rule invariably yields the Condorcet winner as the sequential rule winner, given that there is a Condorcet winner, if each rule in the sequential procedure is a Borda rule.

In 1976, Black [196], one of the earliest of the modern writers to cover the Borda count method, presented an updated justification for the Borda count in this more contemporary paper. He first argued the case for the ordinality, not cardinality, analyses for aggregation of preference orders, both for economists and in politics.

Black defined two models for assigning marks in a Borda count. They are Method I: Accord a 0 mark for a lowest place in a rank order, a 1 mark for a second lowest place, etc. For weak orders, one approach in the event of several alternatives having the same level in a rank order, is to award each alternative the mean of the marks they would receive if they appeared separately and Method II: Assign an alternative 1 mark for each alternative it stands above in a rank order and deduct 1 mark for each alternative it stands below on the rank order. Black then presents his Theorem 1 which says that Methods I and II lead to the same ordering in choice of the alternatives. Due to its ease of use, Black recommends Method II for continued Borda analysis.

Next Black showed that a Borda order may be established from the mean fraction of votes for an alternative or the net number of votes it received. Thus the Borda count provides a convergence between two lines of approach.

Advantages of the Borda count were as follows:

1) It allows one or more minority scores against each alternative to be offset by high majorities scored against others. Condorcet criterion permits no compensation of this kind.

2) Borda takes into account the entire information about elector's performances.

3) Borda yields transitivity.

Disadvantages listed by Black are as follows:

1) It is possible for the alternative with the highest Borda count to collect the high number of votes against a few alternatives and few votes against certain other alternatives.

2) Borda count is an invitation to strategic voting, where voters show their first preference, and then rank the remaining candidates in the reverse of what is believed to be their order of popularity by other voters. This is apt to yield the reverse of the proper order. Black said, finally, that this weakness does nothing where the felt preferences are known.

In 1977, Gillett [71] investigated the indecision probabilities when using the plurality and the Condorcet procedures. The compa.ison did not reach a general conclusion because indecision fluctuates so much. But for small (12 or fewer) even-sized groups, Condorcet indecision is consistently higher than plurality indecision.

In 1978, Colman and Pountney [74] studied the Borda problem, where if there were three or more alternatives, a winning alternative by simple plurality would not be the choice by a majority of the voters. They found that as the number of voters increased, the probability of a Borda problem increased toward 30% with over 300 voters. Correlations with British election results confirmed the theoretical calculations.

In 1978, Richelson [69] conducted a comparative analysis to define a choice function in terms of the maximal set of conditions that the function satisfies. He considered the following social choice functions: (1) Plurality, (2) Borda, (3) Copeland, (4) Dodgson, and (5) the General Optimal Choice Axiom (GOCHA) procedure by Schwartz. The conditions studied are: Majoritarian Independence (MI) where the majority rule relationship between every pair of alternatives stays the same; Independence from Individual Orderings (IIO); Condorcet Principle (CP) where an alternative has a simple majority over every other alternative; Uniform Majority Principle (UMP); Virtually Unanimous Uniform Majority Principle (VUUMP); Majority Rule Monotonicity (MRM); Social Acyclicity (SA); and External Stability (ES).

Of the five functions, only two, (3) Copeland and (5) GOCHA satisfy both MI and IIO. (3) Copeland, (4) Dodgson, and (5) GOCHA satisfy CP, UMP, and VUUMP. Only (3) Copeland satisfies MRM. None of the five methods satisfy SA only (5) GOCHA satisfies ES. In summary, (3) Copeland and (5) GOCHA satisfy five of the seven conditions. When considering dominance and the Pareto principle, (3) Copeland remains as preferred.

As a third paper in a series in 1978, Richelson [70] further examined six voting systems that were all extensions of simple majority rule. Fifteen conditions were considered for the six methods. The methods were: (1) simple plurality, (2) Borda, (3) Copeland, (4) Dodgson, (5) GOCHA, and (6) Black (Condorcet if possible; if not use Borda). The conditions considered are Condorcet Principle (CP); Reduction Principle (RP), Monotonicity (M), Pareto Dominated (PD), Consistency (C), Cancellation (N), Permuted Dominance (PD), Majoritarian Independence (MI), Independence from Individual Ordering (IIO), External Stability (ES), Uniform Majority Principle (UMP), Majority Rule Monotonicity (MRM), Extended Condorcet (EC), Strong Extended Condorcet (SEC), Voter Adaptability (VA), Alternative Adaptability (AA), Duality (D), and Arrow's A3, A4, and A5.

Richelson considered that Conditions IIO and MI (independence); UMP, CP, A5, EC, and SEC (Condorcet), and P (External Stability and Pareto) as most important. On this basis, he considered (3) Copeland and (5) GOCHA most attractive with (2) Borda third. Since (5) GOCHA is less practical, (3) Copeland is considered optimal.

Allen [73] in a paper in 1977, primarily arguing the philosophy of majority rule versus anarchism, stated that the crucial defect in the Borda method of marks lay in the endeavor to assure equality in the distribution of political power by assigning to each elector the same number of marks. He recommended correction by specifying the maximum number of marks an elector may enter in support of any single candidate.

In 1978, Young and Levenglick [77] analyzed Kemeny's distance axiom rule and determined that it is, like the Borda rule, consistent, has alternatives which have a majority over all others, is most preferred, and has all alternatives treated in an unbiased manner.

K. Majority Rule-Multiple Methods Compared

Several papers compared different forms of majority rule methods to determine aggregated preferences. They are summarized in the following paragraphs.

In 1954, Goodman in Chapter 3 of a book by Thrall, Coombs, and Davis, Decision Processes [197], described a specific method presented by Copeland at a 1951 University of Michigan Seminar and a more generalized method.

The "reasonable" Copeland method is based upon the following function of x:

$$s(x) = \begin{cases} +1 \text{ for } x > 0 \\ 0 \text{ for } x = y \\ -1 \text{ for } x < 0 \end{cases}$$

Richelson (68, 1975) effectively restated the s (x) of Copeland to be

$$s(x) = \begin{cases} +1 \text{ for } x > y \\ 0 \text{ for } x = y \\ -1 \text{ for } x < y \end{cases}$$

Next ϕ (i) = Σ s (x) is a measure of row i, and the preferred row is that with ϕ (i) greatest.

As a more general rule, Goodman recommended

$$s(u) = \begin{cases} + \text{ for } u > v \\ 0 \text{ for } u = v \\ - \text{ for } u < v \end{cases}$$

and ϕ (i) provides a utility value if ϕ (i) is positive.

Goodman further stated that, in comparing Black and Coomb's models for aggregation, it is not generally true that the rank order of the median individual will be the rank order obtained by majority rule when single peakedness is satisfied. However, Goodman said, if Coomb's model conditions are satisfied, then the rank order of the median individual is the rank order obtained by majority rule.

In 1966, Svestka [78] in his Master's thesis analyzed Arrow's Theorem, extended Kendall's matrix array of rank orders, and developed a new method he called the Minimum Comprimise (MC) Rule where the "joint order" of the set of alternatives was that permutation of alternatives which yielded the minimum compromise to all individuals in the group. As the number of alternatives increased, the method of determining the partial order of the permutation became increasingly involved. A procedure was developed utilizing an assumption of equidistant separation. Svestka's comparison of the results of the Minimum Compromise Rule and the Simple Method of Majority showed that these methods gave identical results.

Svestka modified the Kendall array to add an aggregating preference matrix. In the sum frequency matrix, the sum of two opposing cells in the array equals the number of individuals in the group voicing a preference for one alternative or indifference for the two alternatives; that cell containing the larger number dominates its opposing. Therefore, the method in the Kendall preference matrix replaces the numbers in all dominating cells of the frequency matrix by unity, and the numbers in all

dominated cells by zero. The rank order is based upon the preference matrix row sums.

In 1971, Haith, [198] developed a probabilistic method to aggregate preferences to evaluate alternative metropolitan water resource plans. Joint political probabilities were computed from the probability distributions of each alternative. The aggregate rank order was determined from the joint probabilities.

In 1971, Castore, Peterson, and Goodrich [81] compared social choice models to the group decision making process to determine if risky-shift was due to the use of an inappropriate model to estimate the group decision. The models compared were (1) the least squares estimate (LS), (2) the modified majority rule (MR), (3) the modified majority rule, considering indifference among alternatives (MR/IN), and (4) the aspiration level (AL) model. The (MR) and (MR/IN) rules use an index V(x) which denotes the number of paired comparisons in x's favor less the number against x (1, 0, -1). The AL model has a satisfaction index of

$$s_i(x_j) = \begin{cases} 1 \text{ if } V(x_j) \geq L_i \\ 0 \text{ if } V(x_j) < L_i \end{cases}$$

with L_i the aspiration level of the ith person. Then the alternatives are ranked according to their value of M_j where

$$M_j = \sum_{i=1}^{n} S_i(x_j)$$

In case of a tied M_j in AL, the larger V(x) in MR/IN is used as preferred.

After tests for 10 three-person and 10 four-person groups, the results were that MR/IN and AL both were significantly more accurate estimates of the group decision than were the LS and MR methods.

In 1973, Wyatt [79] in his Master's thesis compared several rank ordering amalgamating methods to determine the method to best meet his needs for aggregating judgment concerning the influence of reliability and effectiveness factors. The comparison included "Arrow's Method of Majority," "Black's Single Peaked Preference," "Svestka's Minimum Compromise Rule," two feedback methods, a probabilistic method, an additive utilities method, a scaling method, and Thurston's Law of Comparative Judgment.

Arrow's Method of Majority was the only method which had favorable classification for all requirements. It did not require that the rank orders be cardinal utilities, transitive orders, complete orders, strong orders, restricted number of judges or alternatives, or dependent rankings. The rank order can be distribution free. Wyatt's definition

of Arrow's Method of Majority was quite similar to Svestka's extension of Kendall's Array Matrix method, with the aggregating frequency matrix transitioning to a (1, 1/2, 0) preference matrix. Wyatt explains how this method of majority satisfies Arrow's definition, but does not establish that it uniquely satisfies Arrow.

In 1974, Chartier and Wertheimer [80] compared six election schemes through simulations. The six schemes were majority, plurality, weights, single transferable vote (STV), which transfer votes from early losers or winners to those still in contention, satisfaction (SAT) which allows a vote to assign preferences of his own choosing, and electorial. Except for majority, the schemes picked the same winner. Majority was not comparable and a different winner was chosen frequently. This paper had finite conclusions but provided too small a basis to substantiate the findings.

In 1975, Young [199] studied and reported conditions for social choice functions and reported the family of functions satisfying these conditions. The family of functions considered was plurality and Borda functions. Only the Borda function was shown to be neutral, consistent, faithful, and having the cancellation property; all of which are desirable properties.

Richelson, 1975 and 1978, [68, 69, 70] (summarized elsewhere in this survey), comparatively analyzed six social choice methods with respect to 18 conditions. He concluded that Copeland, GOCHA, and Borda are the most attractive functions in that order. GOCHA is a General Optimal Choice Axiom, proposed by Schwartz.

L. Transitivity, Intransitivity, and Cyclicity

Many papers have stressed the probability of intransitivity and the conditions to maximum transitivity when aggregating individual rank orders. The papers will be discussed in the following paragraphs.

In 1953, May [37] substantiated that intransitivity is a human nature phenomenon and must be considered in preference analyses. The conditions for transitivity failure must be determined. He further demonstrated that intransitivity may occur in individual preference choice rank orders or it may occur in aggregated rank orders where the aggregation was made from transitive individual orders.

In 1958, Davis [200] investigated the question of whether circular triads in an intransitivity preference rank order are stable or random. His analysis of the literature and two experiments, one like May's student wine preference experiment and one like Edward's student betting preference experiment, both showed no evidence to demonstrate that stable circular triads exist. He stated that evidence on the stability of circular triads is essential before one needs to deal with the probability of intransitive preferences.

In 1964, Sen [201] studied the handling of transitivity of majority decisions in the writings of Arrow and Black. Black's works are for choices by votes, while Arrow's are arguments in terms of preference rather than votes. Indifference to Black is an abstained vote while indifference to Arrow's system means that utility votes are equivalent, but not necessarily zero. Sen concluded by stating that with Black's method of majority decision, transitivity is not guaranteed by single-peakedness, whereas in Arrow's method of majority decision, the decision is not generally observable from voting statistics.

In 1964, Inada [202] developed a most comprehensive set of simple majority decision rules to aviod intransitivity. For the simple majority decision rule to be always transitive Inada said the society must consist of voters whose preferences among any three alternatives are at least one of the following types:

1) If the number of voters is even or odd (free):

a) Dichotomous preferences-which means $\xi P \eta P \zeta$ is not in the list of possible individual orderings.

b) Echoic preferences-which mean if $\xi P \eta P \zeta$ is in the list, then ξ is "best" or ξ is "worst" for any other ordering in the list.

c) Antagonistic preferences-which means $\xi P \eta P \zeta$ is in the list and $\zeta P \xi$ or $\xi I \zeta$ is the order for any other orderings in the list.

2) If the number of voters is odd:

a) Value-restricted preferences-which means all of the following are present. Transitivity also occurs if, for odd numbers of voters, any one of the following exist:

(1) Single-peaked preferences conditions are satisfied.

(2) Single-caved preferences conditions are satisfied.

(3) Alternatives are separated into two groups.

b) Taboo preferences-which means xIyIz is not in the list, and x is "best" or y is "worst" for any ordering in the list (or both xIyIz and yPx are not in the list).

Inada also went further and provided similar conditions for strong orderings.

In 1970, Pomeranz and Weil [203] computed the probabilities of having a cyclical majority (winner) for 3 to 40 issues (alternatives) and 3 to 37 judges (oddnumbers only). Their values compared well with prior literature where available for the smaller values (up to seven

issues). The probability of a cyclical winner is 0.80 for 37 judges and and 40 issues but is only 0.09 for 37 judges and three issues. For three judges and 40 issues the probability is 0.61; establishing the higher sensitivity to the number of issues.

Their method uses a preference matrix like Black's Borda examples, or Shannon's except that the rows and columns are reversed, so a no-majority case is determined when all rows have an entry in the summation preference matrix greater than half the number of judges. This could be determined in the Shannon summed preference matrix by identifying cases with all rows entries less than half the number of judges. The probabilities of cyclical majority apply only to the winner in each aggregation, not to cyclical triads lower in the rank order.

In 1970, Fishburn [83] theoretically extended the results of Sen and Pattanaik on sufficient conditions for the transitivity of simple majorities to the case where it is not assumed that the individual's indifference relations are transitive. He also applied the theory to the indifference plateau definitions for single peaked analysis as developed by Black and Arrow.

In 1970, Fishburn [191] presented new examples to support his contention that individual social transitivity was not a fair requirement for every social choice function, as was one of Arrow's conditions in his impossibility theorem. One example was based on choosing transitive alternatives which cannot be implemented. Other examples attacked the transitivity of individual indifference as an absolute rule. In both cases Fishburn argued for flexible application of individual transitivity.

In 1971, Sen [86] systematically developed the axiomatic structure of the theory of revealed preference. Specifically, he covered the constraints on preference orders to achieve required boundaries, element versus set valued choices, and the binary character of the conditions that guarantee transitivity.

In 1972, Davis [31] established a new formulation of the calculus theory of social choice based on dominance, unique medians which provide transitive and complete orders. The theory and method were consistant with Arrow's Impossibility Theorem.

In 1972, Bowman and Colantoni [204] presented a synthesis of the restrictions to insure a transitive group preference. The equivalence of transitivity and the extended Condorcet condition were shown. The mechanism of a system of linear inequalities was used. The Latin Square analysis of cyclical triads was applied. Inada's conditions and Black's single peakedness conditions were shown as bases for the extended Condorcet condition.

In 1972, Jamison and Luce [205] developed a new approach toward intransitivity that neither left the probability distribution on preference orderings unchanging nor allowed a large number of

probabilities to go to zero without good evidence. The method was based on how homogeneous the society was assumed to be. A term σ, which is the index of social homogeniety is the basis to analyze the probability of intransitive majority rule. As σ decreases, the probability of intransitivity decreases. Monte Carlo tables are prepared to give the probabilities of intransitivity for three alternatives for number of voters up to 15 and σ values up to 126. In this table, the probabilities of intransitivity vary from 0.0013 to 0.0919.

In 1972, Rao [206] showed that under more general conditions of six different orderings: xPyPz, xPzPy, xIzPy, xPzIy, yPxPz, yIxPz, yPxIz, and xIyIz; the simple majority rule will always be transitive provided the number of individuals is odd. The six possible triad cases represent all possible combinations of weak orderings over a triple.

In 1973, Kramer [207] extended the various conditions for non-transitivity of majority rule into multi-dimensional space. When voter preferences can be represented by quasi-concave, differentiable functions, the various transitivity conditions are not significantly less restrictive than the extreme conditions of individual preferences.

In 1975, Jamison [84] used empirical data to try to prove the inadequancy of theories that restricted the domains of individual preferences to obtain transitivity. The experiment was run with two groups of students: 67 in one group and 42 in the other. Each random sample consisted of 4500 draws, each from the data base of the student's transitive individual rank orders. The results indicated the following:

1) As the number of voters increased from three to five, the probabilities of an intransitivity tended to rise slightly or remain constant; as the number of voters increased from five to fifteen, these probabilities tended to decrease monotonically from 50% to 75% of their values at five voters.

2) As the number of alternatives increased from three to six the probability of an intransitivity increased by a factor of approximately ten.

In 1975, Saposnik [85] developed additional conditions for transitivity for strong orders and quasi-transitivity for weak orders. The condition, cycle balance (CB), is satisified if the number of individuals having rankings constitute each of the cycles. His first theorem states that under simple majority rule, a rank ordering is transitive if the ordering satisfies a CB. The second theorem, like the first, states that for reflexive weak orderings if CB is satisfied, the simple majority rule rank ordering is transitive.

In 1975, Kaneko [208] established conditions for transitivity for a rank ordering as developed by two types of decision processes: a dominance in a proper simple game and aggregate preference by the simple majority decision rule. For both theories, the transitivity condition

was of the form of one relation, i.e., xPy dominating in the game in all individual orders or xRy prevailing in the majority rule aggregation for all individual rank orders.

In 1976, Merchant and Rao [209] extended the Bowman and Colantoni decision function, $d^k(P^*, P)$ into a set covering problem which was proven acyclic and thus a mechanism to obtain transitive aggregate rank orders. They also developed an algorithm to minimize tha maximum distance $|P_{ij}^* - P_{ij}|$ between all pairs of points, thus solving the bottleneck quadratic assignment problem. Any transitive ordering obtained by the algorithm was proven to represent an optimal solution.

In 1976, McKelvey [210] developed for only the case of Euclidian metrics, that if there were intransitivities, these intransitivities extended to the whole space in such a way that all points were in the same cycle set. Thus it was theoretically possible with perfect knowledge of everyone else's votes, to design voting procedures which could end up at any point in the space of alternatives. This was developed through the development of a Condorcet point which was a strong total median and this resulted in a transitive rank order.

In 1978, Grandmont [174] developed the theory to develop intermediate relations such that rank orders are transitivite. Further, Grandmont showed that the concidence of the majority rule with the intermediate order hold even when the relations are not transitive. Finally, it was shown that single peakedness was a one-dimensional form of the solution of a multi-dimensional family.

M. Basic Arrow and Majority Rule Theory

Various authors have amplified Arrow's aggregate social choice theories. These are highlighted in the following paragraphs.

Luce's paper [87] was included in the 1971 book edited by Arrow Selected Readings In Economic Theory From Econometrics, the MIT Press, Cambridge, MA, 1971. Luce developed a new set of axioms for preference theory based on the assumption that indifference preference relations need not be transitive, but may be intransitive. He developed the term "semiorder" to define relations satisfying the intransitive indifference axioms. Most of the article then developed thorough semiorder theory.

In 1964, Koopman [92] published his article on future preference theory in the book edited by Shelley and Bryan, Human Judgments and Optimality, John Wiley and Sons, Inc., New York, 1964. Koopman emphasized the difference in the analysis of "preference" and "opportunity." Preference was defined as an ordering of an all inclusive set of alternatives, while opportunity consisted of those alternatives actually under the control of the judge. Koopman developed the theoretical philosophy to include preference ordering of opportunities.

In 1964, Inada [40] developed a theory for two additional conditions to extend Arrow's theory. One condition added was that a social welfare function (SWF) is possible if all alternatives can be classified into two groups so all individuals are indifferent among alternatives in each group. The other condition permits an SWF if the preferences are single-peaked and the number of individuals is odd.

In 1969, Inada [202] published a comprehensive theory and list of all ordering restrictive conditions that must be met for attaining a transitive aggregated social ordering. The list included dichotomous preferences, echoic preference, antagonistic preferences, and taboo preferences.

In 1969, Fishburn [211] developed a theoretical condition of social choice ordering for weak orders to be preference summable. Fishburn showed that the Borda method was preference summable for $m \geq 4$ but that Copeland's method was not when $m \geq 6$.

In 1974, Fishburn [89] developed the revised conditions of Arrow's Impossibility Theorem to determine when a social welfare function SWF is impossible if the individual preference orders are not complete by including all possible pairs of alternatives.

In 1974, Fishburn [88] developed the theory that there can exist strict social orders aggregated from complete and partial orders which can cycle, due to the presence of the partial individual orders.

In 1975, Cole and Sage [212] presented a comprehensive coverage of the theory aggregation of individual preference orders. This emphasized Arrow's Theorem and several restrictions on orderings to permit social welfare functions. In addition the concept of hierarchial group decision making was developed where aggregated decisions in a subgroup were analyzed as individuals in the group at the next higher level. The Bowman and Colantoni method to force transitive majority rule was studied in detail.

In 1977, Fishburn [90] developed a comprehensive theory concerning using binary relations for decision making. The paper presented theory for ordinal and cardinal binary relations.

In 1978, Grandmont [91] developed the theory for intermediate preference where the preference function magnitude was between two other preference functions. The emphasis was to accept all orders but focus attention on the distribution of the individuals.

In 1978, Fishburn [213] developed the theory of ordinal preferences where the individual judges vote on, not rank, the alternatives. A judge may cast one or more votes but all are equal. Fishburn further searched for any strategic theory that might make multiple alternative, multiple vote per judge to be most fair.

N. Majority Rule-Minimum Loss Methods

Majority rule models have been developed to aggregate rank orders which utilize the concepts of minimization of individual perceived losses and median techniques. These models are described in the following paragraphs.

In 1960, Van den Bogaard and Versluis [93] developed a technique to aggregate individual welfare preferences by minimization of the social loss function which is assumed to be a linear combination of individual loss functions. Individual loss functions are defined corresponding to welfare functions. Each loss function specifies the welfare difference associated with optimum policy and an alternative policy. The paper demonstrates its loss function technique by application to the 1957 Dutch economy.

In 1963, Theil [96] extended the theory and details of Van den Bogaard and Versluis in developing the concept of aggregation by minimizing the combination of individual social loss functions.

In 1975, Hoyer and Mayer [95] stated that their purpose was to generalize the model of Davis, et al. by allowing each judge to define his social loss with repect to the j th alternative in a manner unique to his own interpretation of the relative importance of each issue. The loss for each judge was defined by a quadratic function. The loss minimization aggregation approach included graphical analysis.

Hoyer [94] in his dissertation (1977) continued loss minimization analysis for preference aggregation. Graphical explanations were used throughout. His model was compatible to a spatial formulation and was analyzed under seven important objective functions defined by Hoyer. Three functions are expected plurality, proportion of expected votes, expected votes, probability that plurality exceeds some level, probability that proportion exceeds some level, and expected proportion of the vote. Hoyer's method accounted for indifference and abstention.

O. Survey Literature

Several books and papers broadly surveyed the state of knowledge for aggregation of rank orders of preferences. For some the reference covered only a survey of what others had published, while others presented a survey as a basis before making a report on their own more narrow research conclusions. In chronological order, the highlights of the references with regard to aggregation of rank orders will follow.

Rothenberg's book, 1961, [103] contains a section which covers Arrow's Impossibility Theorem and how it relates to Bergson's welfare economics theories. Rothenberg's definition and explanation of single peakedness is excellent and more directly applicable than most other writers on the subject. The book contains social choice and social welfare needs but few practical, empirical models.

In 1961, Riker [98] presented an early coverage of the area. His paper presented a thorough bibliography beginning with D. Black's papers of the late 1940's. Riker continued with an explanation of the Paradox problem (i.e., a > b > c > a) and the 18th and 19th century writers that first identified the paradox and attempted to develop techniques to avoid its effects.

Next, Riker presented Arrow's Impossibility Theorem, and the comments of related papers by researchers such as May, Luce and Raiffa, Inada, and Murakami. These other writers analyzed and reworded Arrow's conditions for a social welfare function. Riker further interpreted Arrow's Theorem in a political context. In striving to achieve a usable aggregated rank order, Riker presented the approach of using an odd number of judges single-peaked individual orders, or the unfolding technique of Coombs. To help quantify the problem, the probabilities of intransitivity and triplets were determined. In summarizing, Riker emphasized Arrow's and Black's influence on political theory.

In 1964, Coomb's [97] included considerable information on aggregation of ordinal rank orders in his book on data theory. He developed his unfolding technique using the property of stochastic transitivity. He also presented alternate theories of preferential choice including Black's single-peaked preference function and lexiographic models by Hutt and Young and Greene. Coombs also presented Arrow's Impossibility Theorem and its conditions. He later showed several examples of triangular analysis.

In 1966, Lazarsfeld and Henry [214] as editors included an essay by G. T. Guilbaud who began by surveying the aggregation related writings of many French researchers. He described the Condorcet paradox effect, reviewed Condorcet majority rule aggregation method, and demonstrated the logic for ordinality rather than cardinality in preference order aggregations. Guilbaud concluded this area by contending that the only universal rule for aggregation is for society to elect a dictator.

Sen's [99] survey book presented a most comprehensive coverage of rank order aggregation theory, beginning with the binary relations, properties, and theory. Then Sen covers, most thoroughly, the same areas covered by Riker described previously.

Cochrane and Zeleny [101] as editors presented two papers on aggregation theory. One, by J. M. Blin discussed aggregation consistency. The second, by B. Roy, discussed outranking relations to analyze an aggregation or to obtain a weak order.

A second most comprehensive coverage book is by Fishburn [100] who covers in a practical vein, all of the topics of Riker and Sen.

In 1974, Cotter [100] as editor included an essay on Theories of Collective Choice by K. A. Shepsle, where Shepsle primarily focuses on theories of political choice through examination of the structure of

choice, rational behavior, and collective decision making. Next Arrow's works are described. Little space is devoted to empirical studies of collective choice as Shepsle (and this writer) regarded the virtual absence of empirical data a major weakness of the collective choice literature.

In 1975, Brown [104] surveyed the problem of aggregation of the preferences of individuals into a social preference relation. He surveyed the impact of Arrow, Pareto, Condorcet, and Black. Brown then presented a voting procedure with an agenda for a sequence of voting to eliminate one of each pair compared. Using Plott's work, Brown established the effect of path-independence on cyclical majorities.

Seaver [215] surveyed aggregation theory as part of the introduction to his report. He covered the works of Arrow including his conditions, Fishburn's writings, Dalkey's work on probabilities, Pattanaik's distinction between a social decision function and a social welfare function, and Dalkey's concept of anchored ordinal preference scales.

Plott [216] in a comprehensive recent workshop paper, presented the usual description of the voting paradox, Arrow's Impossibility Theorem, social preference principles, and axioms for transitivity. In addition, Plott describes several aggregation ordering models such as the majority rule, Borda's model, a competitive pricing model, a binary process, and cooperative games models.

Sticha [217] in his dissertation, defined and discussed additive and non-additive models. Additive models were based on theories by Mays, Black, and Fishburn, as well as the Borda majority-rule method. The non-additive models includes work by Murikami, Fishburn, and Shapely.

Cohon [102] in his multi-objective programming and planning book, presented various weighting concepts to be used in aggregation models, welfare economics, and Arrow's Theorem, including the writers who disagreed with Arrow's great work. Cohon considered that Sen, as described previously, wrote a book to verify aggregation concepts and theory. Cohon also presented the game theoretical approach to aggregation.

P. Majority Rule-Game Theory and Strategy Proofness

Using the work of von Newmann and Morgenstern (1947), several writers have adapted game theory to the concept of the simple majority rule for analyzing the aggregation of individual preferences. Several of these papers are discussed in the following paragraphs.

In 1961, Barbut [105] discussed majority rule, Condorcet's paradox, and introduced the philosophy of the relationships of game theory in its elemental form of a two-person, zero-sum game. Von Newmann's Theorem is

on the affirmative existence of the equilibrium of the strategic playing in any two-sided game. Barbut introduces but does not complete the relation of preferences aggregation and game theory.

In 1974, Shishko [106] presented some of the game-theoretic solution concepts applicable to n-person majority-rule games. Definitions are established for side-payment cores, imputations, dominance, and balanced sets. He shows that the core of an n-person simple majority game must be empty. The game theory application permits players to pick allocations theoretically to leave them better off.

In 1975, Nakamura [108] generalized the theory for a sufficient condition for an arbitrary proper simple game with ordinal preferences to have a nonempty core. This occurs without side payments and transferable utilities.

In 1975, Buckley and Westen [107] discussed all known von Newmann-Morgenstern solutions to the majority rule game (n, k) where $m/2 < k < n$ are included. Experimental results for the (4, 3) and (5, 3) games were discussed.

In 1976, Buckley and Westen [218] continued their majority rule game theoretical work to include comparison for "Kernels" and "bargaining sets" as predictors of experimental trial games with four or five persons. They were found to be good predictors.

McKelvey [210] extended game theory to symmetric spatial games where majority rule equilibrium is not present. The method utilized mixed minima strategies.

Salles and Wendell [109] extended the game theoretic work of Nakamura (1975). When every individual had a strictly quasi-concave utility function over a closed bounded interval, then it was shown that the local core in a proper simple majority rule game was precisely the core.

In a somewhat different route, but related to the objective of game theory in individual voting, is the work to develop a theory to manipulate the results of the aggregation. One writer on this subject is Gibbard. In 1977, Gibbard [110] emphasized the theory involved in voting by rank order ballot where individual manipulation is considered. From this, Gibbard developed the criteria to make a voting scheme strategy-proof.

In 1979, Rubinstein [219] presented a more general analysis of the core characteristics in a majority rule game. He used topology theory in the analysis.

Q. Tullock's Books

Gordon Tullock has written controversial, critical material concerning the usefulness of Arrow's work in this field and concerning

the majority-rule group decision method. Several books and articles which cover Tullock's claims and several responses are highlighted in the following paragraphs.

Buchanan and Tullock, 1962, [111] described the author's impressions of political organizations and analytics of a society of free men. Many subjects were covered, but specifically on the field of social choice analysis, the book severely criticizes Arrow's 1951 book and theorems as being outdated and not useful. Black's book containing single-peaked preference analysis methodology and excerpts from historical works by author's such as Borda and Condorcet were highly commended. In the area of majority rule techniques, the book is generally critical of techniques that have been offered since Black's effort.

In 1967, Tullock [112] analyzed the social choice research of the prior fifteen years and presented his propositions. First, although mathematically sound, Tullock stated that in most cases Arrow's Impossibility Theorem "will seldom be of much importance." Tullock adapted Hotelling and Down's works on political decisions. Duncan Black's single-peak preference curve analysis was commended as highly useful. Tullock continued his book with quasi-philosophical analyses of the interactions of economics and politics.

In 1969, Arrow [113] reviewed Tullock's book which was critical of the usefulness of Arrow's Impossibility Theorem. Arrow's review is factual, subtle, generally positive, and professional. He added a proof to an analytical example Tullock discussed to show that under certain conditions, the Condorcet voting paradox would not arise.

Barton [115] analyzed and extended the work of Buchanan and Tullock concerning the choice of a selection rule as a result of individual interaction. Barton accepted Buchanan and Tullock's model, but improved it through three differences:

1) A plurality rule does exist.

2) Simple majority is a special rule.

3) Simple majority will be adopted with great frequency.

In 1973, Tullock [220] responded to Barton's paper on Buchanan and Tullock's book. Barton had interpreted that Buchanan and Tullock opposed the concept of voting rules in which less than a majority vote decision would be permitted. In this paper, Tullock emphasized and explained why he believed less than majority rules are necessary, viable, and realistic considerations where more than two alternatives are to be voted upon.

In 1973, Hanson's [114] dissertation critiqued the Buchanan and Tullock book [111] especially concerning the utility of majority-rule

as a social choice rule. More specifically, Hanson's objective was to provide a critical review of the claims of Buchanan and Tullock and to offer an alternate model to "analyze the major problems of collective decision-making." The most objectionable claim by Buchanan and Tullock was that "majority rule allows a winning coalition to secure handsome net gains by imposing significant net losses upon the losing coalition." Hanson said the consensus of the critiques of Buchanan and Tullock was that they "omit the type of proposal which is generally enacted by actual decision-making groups." Hanson's research concluded that Buchanan and Tullock's claims concerning majority rule are valid only under three restrictive conditions:

1) Majority rule can only have nonproductive outcomes if the yield has total benefits less than the total cost of the outcome.

2) The decision making group is very small.

3) Each majority rule outcome proposal is "like a private good because it yields benefits to some specific group member." Hanson further contended that Buchanan and Tullock are wrong in that "Arrow's result still stands as a valid analysis of a critical limitation of majority rule."

Hanson showed "that with a slight variation in the conditions postulated by Buchanan and Tullock, different conclusions can be reached concerning the effects of decision-making rules." An example is that the group size not be small. An alternate model was developed to overcome the deficiencies of Buchanan and Tullock's work.

R. Majority Rule Examples

Several papers discuss examples of applications of the majority procedure to real decision problems, expecially concerning management.

In 1963, Ward [118], developed an analysis of firm decision making utilizing group or committee action. For simplicity, firm hierachic nature was replaced by a single board. The lists of member's preferences among the alternatives were aggregated by an Arrow-type of simple majority rule social welfare function. Single-peakedness and Latin squarelessness analyses were used to evaluate the consistency of the aggregated ranking. When used for price policy determination, the aggregation by majority rule did not lead to reliable predictions as to which outcome would be chosen by the firm because of the difficulty of a clear decision of group rankings, because of rankings of the wrong sets of alternatives, with relation to the overall questions, and because majority rule can deviate from the idealized process.

In 1970, Shannon and Biles [117] presented the results of a statistical survey conducted to determine the utility to operations research practitioners of curriculum topics. A national survey obtained a rank-order listing of topics that are commonly found in Master's Degree

programs. The lengths of the listing of topics varied. Using Arrow's majority-rule method, as specifically proposed by Shannon in 1968, the individual rank orders were successfully amalgamated into a single rank order preference.

In 1975, Campbell [116] studied the socio-economic effects of national income distribution using a simple-majority rule technique for the process of redistributing after tax income. The policy yielded indications of the power of the coalition of poor voters to ameliorate their lives by voting for transfers of wealth. Apparently, the poor have more power under fair taxation than under progressive taxation.

S. Voter's Paradox

The voter's paradox has concerned analysts since Condorcet identified its presence. Arrow established the paradox as a primary characteristic in his Impossibility Theorem. Since Arrow, several researchers have strived to quantify the probability of a voter's paradox situation occurring thereby quantifying the practical significance of Arrow's Theorem.

In 1965, Campbell and Tullock [221] worked to continue the work begun by D. Black [8] when he included a table on the proportion of cyclical majorities that could be expected. A cyclical majority occurs when the multiple winners repeat themselves. If the percentage of cyclical majorities is large, Arrow's Impossibility Theorem would be practical as well as theoretically significant.

A computer model with Monte Carlo simulation of a committee voting was used. For each data point, at least 1000 cases were calculated. For example, the largest value for 19 voters with 17 issues is 62.6% cyclical majorities. In summary, with independent preferences, the cyclical majority is clearly an important phenomenon, and therefore Arrow's Impossibility Theorem is not a trivial one.

In 1966, Klahr [122], like Campbell and Tullock, strived to satisfy Black's quest for a generalized analytical expression for the probability of intransitivity. As an empirical approach, Klahr developed a computer model to compute the probabilities for cases up to six issues and seven judges. His values compared favorably with Campbell and Tullock. For example, he concluded that a committee of three judges is three times as likely to reach an intransitive impasse when they consider five issues (alternatives) as when they consider three issues. However, increasing a committee size from three to five members has a relatively small effect on the probability of an intransitive rank ordering. This paper also demonstrated that equally likely individual orderings do not aggregate to equally likely rank orderings.

In 1968, Garmon and Kamien [120] developed an analytical expression to determine the frequency of the "voting paradox" as a function

of the number of voters. They used the definition that each voter's rank was

$$v_{ij} = \begin{cases} 1 \text{ if } x_i > x_j, \; i \neq j \\ 0 \text{ if } i = j \\ -1 \text{ if } x_j > x_i, \; i \neq j \end{cases} .$$

The aggregate matrix is

$$Q = \sum_{t=1}^{n!} r_i \, V^t$$

and the probability of a voters paradox (no winner) is

$$P(m, n, \bar{s}) = \sum_{(\bar{r} \epsilon R)} \left(r_1, r_2^{\,m}, \ldots r_{n!} \right) \prod_{t=1}^{n!} s_t^{rt} \quad ,$$

where m is the number of judges, r is the number of each type of vote, and s is the probability that a judge is from a culture that will cause him to select a vote in a certain manner (r). This expression was well correlated to Guilbaud's number:

$$\lim_{m \to \infty} P(m, 3, \bar{s}') = 0.0877$$

and a table was calculated for values of n (number of alternatives) up to 8, where

$$\lim_{m \to \infty} P(m, 8, \bar{s}') = 0.415 \quad .$$

The value for n = 6 was

$$\lim_{m \to \infty} P(m, 6, \bar{s}') = 0.315 \quad .$$

In 1968, Niemi and Weisberg [123] also developed a general analytical expression for the probability of intransitivity in an aggregated rank order. Their general expression is

$$P(m,n) = 1 - \sum_{i=1}^{n} Pr \left\{ A_i > A_j, \; \forall_j \neq i \right\}$$

where

$$\Pr\left\{A_i > A_j \; ; \quad \forall_j \neq i\right\}$$

is given by

$$\sum\left(\begin{matrix} m \\ u_1 \ldots u_{n!} \end{matrix}\right) \; P_1^{u_1} \ldots P_{n!}^{u_{n!}}$$

where u is the number of voters with a given rank order. Again the

$$\lim_{m \to \infty} P(m, 3) = 0.0877$$

and

$$\lim_{m \to \infty} P(m, 6) = 0.315$$

In 1973, Weisberg and Niemi [124] extended the theoretical expression for the probability of intransitivity by using a pairwise probability approach. This method can be used without knowing the probabilities of the individual rank orders. The pairwise probabilities suffice. All that need to be specified is whether each pairwise value is less than, or equal to, or greater than 0.5. Limit values in this resulting data table agree with prior references.

In 1976, Gehrlein and Fishburn [121] extended the analysis of the probability of a Condorcet paradox to consider anonymous profiles (A-Profiles), which is a function that assigns a nonnegative number of voters to each potential preference order on the alternatives such that the sum of the assigned integers equals n. The results, calculated for three alternatives, consistently showed lower probability of paradoxes for equally likely A-profiles than for equally likely profiles. The difference is approximately 2% for m = 3 ($\sim$ 6.25% versus 8.7%) and is approximately 4% for m = 4.

In 1976, Abrams [119] determined that increased homogeneity of individual rank orders can, in certain cases, increase the paradox probability.

T. Majority Rule Conditions and Equilibrium

Since Arrow and Black, many writers have developed further conditions necessary for a majority rule aggregation of individual preferences to result in an equilibrium situation where the final aggregated rank order has a winner and is probably acyclic. The papers to be described build up this wealth of theory.

In 1966, Murakami [125] presented a set of conditions for a simple majority decision. The majority voting operation must be nondictatorial.

The majority voting operations logical constants are 1, 0, -1 and must have Condition A (autonomy), and Condition B (nonreversal), as well as Condition C (nondictatorship). A self-dual function is one where $F(-D_1, -D_2, \ldots) = -F(D_1, D_2, \ldots)$. In conclusion, a group decision function is a majority decision only if the function is self-dual, monotonic, and nondictatorial.

In 1970, Pattanaik [126] presented general theory on the existence of a choice set by a majority voting system. The conditions of value restrictions for not-worst (NW), not-strictly-best (NSB), and not-strictly-medium (NSM) were defined as restrictions relating to the existence of a choice set. The choice set exists if a set s satisfies the NSW value restrictions, the NSB value restrictions, or the NSM value restrictions.

In 1972, Fine [127] gave necessary and sufficient conditions for a social decision rule for two alternatives to be representative. The conditions involved monotonicity, faithfulness, and zig-zag.

In 1972, Hinich, Ledyard, and Ordeshook [222] developed an analysis of conditions for a majority decision. They also emphasized the conditions and effects when a segment of voters abstain and the strong influence the abstaining voter can have on a majority rule decision.

In 1973, Singamsetti [223] examined a wide range of conditions for the existence of a social preference function by the method of simple majority rule. A more general *set of leader-follower* preference conditions are proposed in this dissertation.

In 1973, Windell and Thorson [224] proposed a generalized theory of optimal decisions under majority rule which was based upon assumptions less restrictive than those in previous papers in this area. They defined various kinds of equilibrium (or dominant) points in the literature, then characterized a general class of indifference contours which are equivalent to norms' functions. Loss functions were formulated to allow for various norms that depend upon particular citizens. If all citizens used the same arbitrary norm, then the median would be an equilibrium point.

In 1974, Nitzan [225] formulated various conditions on individual preferences, which guaranteed the existence of equilibrium under majority rule. The conditions divided into two categories: the exclusion and the symmetry conditions. The paper showed that if individual tests were assumed to be characterized by lexicographic orderings, there were still cases in which an equilibrium was assured. He showed that when preferences of individual voters were defined on a two-dimensional space and satisfied lexicographic order, as defined by Nitzan, a majority-decision rule is rational as an equilibrium always exists.

In 1973, Kelly [226] refined the concepts of necessity conditions in voting and applied them to yield quasi-transitive and transitive social preference relations.

In 1976, McKelvey and Wendell [227] examined conditions for the existence of voting equilibrium over a multi-dimensional issue (alternative) space. Many local theorems were extended to be global for majority rules. Majority and plurality Condorcet points and cores were related in this theoretical analysis.

In 1977, Blau and Deb [128] showed that social decision functions (SDF) produced acyclic social preferences if the SDF was of a veto nature, had individual indifference, and was finite. When the SDF was infinite, no choice was possible. The original United Nations Security Council is a good example of a neutral, monotonic SDF with a veto hierarchy.

In 1977, Richelson [129] developed a comprehensive classification of essentially all conditions that the literature has identified for a social choice function. This work is most thorough, and as the author hopes, it should help systematize research in various areas of social choice. To list all conditions would be repetitious. As as example, the first grouping under ethical conditions is Pareto Conditions which is composed of Binary Weak Pareto (BWP), Binary Strong Pareto (BSP), Weak Pareto (WP), Strong Pareto (SP), Reduction Principle (RP), Semi-Fairness (SF), Fairness (F) conditions. Each condition is defined by Richelson. The other ethical groupings are Condorcet conditions, transitive closure of M conditions, cancellation, permuted dominance, and symmetry. Aggregation conditions contain groupings entitled Independence conditions, responsiveness and monotonicity conditions, anonymity; neutrality, and duality, and separability and elimination. Rationality conditions contain groupings entitled collective ordering conditions, base relation conditions, revealed preference conditions, subset choice conditions, path independence conditions, extension conditions, and game theory and revealed preference conditions. For all of the types of groupings, Richelson provides simple logical arrow diagrams to relate equivalent and derivative relationships between the social choice functions.

In 1978, Peleg [182] constructed and examined exactly and strongly consistent anonymous social choice functions and investigated various properties. The exactly consistent class of social choice functions (voting schemes) is not distorted by manipulation and strategic voting. Quantitative conditions are for $n \geq m - 1$ and $K(n,m) = [n(m-1)/m] + 1$ for SCF condition W_k, the winning condition for every k person coalition. It can be shown that for $n = 4$, $m = 3$, for example, no exactly and strongly consistent SDF exists.

In 1978, Rader [228] developed theory to use possibly intransitive and incomplete preferences to obtain and induce preferences for economic trading and consumption.

In 1978, Sengupta [229] presented an approach to analyze strategic voting and the possibility of sincere voting. The results showed that there is no guarantee that individuals would always vote sincerely, even with extreme caution to prevent changing from sincere voting. Therefore, an analysis must be made as to the suitability of given strategy rather than given situations only.

In 1979, Shepsle [230] extended the conditions for equilibrium in multidimensional social choice to those of institutional arrangements such as committee systems, juridictional arrangements, and amendment control rules. The principal thrust of the paper was the ways institutional arrangements might conspire with individual preferences to produce structure-induced equilibrium.

U. Weighting Methods

When aggregating sets of rank orders, it is frequently necessary to apply weights to the rank-order elements to account for variations in the importance of the alternatives to the decision maker, the importance of a single or group of judges to the decision maker, or the self-expertise appraisal of individual judges. The application of weights for the above reasons will tend to make the single aggregated output rank order more fair. A few representative articles covering the available methods of applying the types of weights mentioned previously have been identified and summarized in the following paragraphs.

In 1968, Winkler [130] compared several weighting schemes for several consensus gathering methods. The general method is

$$f(\theta) = \Sigma_i w_i f_i(\theta)$$

where $w_i \geq 0$

and $\Sigma_i w_i = 1$.

The $\Sigma_i w_i = 1$ is required to normalize $f(\theta)$. The various forms of w_i studied are:

1) Equal weights, where the judges cannot differentiate.

2) Weights proportional to a ranking.

3) Weights proportional to self rating

a) As a w_i term, continuous from $0 \rightarrow 1$

b) As a go-no weight to determine to exclude the alternatives ranked.

Weights are based on comparisons of previously assessed distributions with actual outcomes. Winkler concludes the paper, *after comparisons of* weights and consensus methods, that the consensus methods have effect but not the type weight.

In 1971, Gustafson, Pai, and Kramer [133] developed a weighted aggregate approach to R&D project selection. Their method had two steps for the decision makers to take:

1) Rank, in order of importance, the related criteria for choice of R&D projects.

2) Assign a value of 100 to the most important criteria and values between 0 and 100 to the others to reflect their relative importance. Then these weights are normalized and successively multiplied by weights of related criteria at each higher level.

In 1972, Chernous'ko [131] developed a weighting scheme in which the experts characterize their own expertize in the problem under consideration. The weight used c_i is ≥ 0 and $\Sigma_i c_i = 1$.

In 1972, Klee [132] concluded that to combine expert opinions, the following weights might be used:

1) Equal weights.

2) Weights based upon previous performance of the expert.

3) Weights proportional to a self-rating.

Rowse, Gustafson, and Ludke [134] analyzed aggregation rules for combining likelihood ratio estimates of experts. The method of weights studied were:

1) Equal weights (EW).

2) Peer weights (PW) where each judge's estimates are weighted by the other judge's ratings of the first judge's overall estimation ability.

3) Self-weights (SW) where each judge weights each of his estimates.

4) Group weighting (GW) where all judges weight each judge's overall estimation ability, then the weights are normalized and summarized.

5) Average weights $\left(AW = \frac{SW + GW}{2} \right)$ which is a variation of the group estimation ability. The authors concluded that more refined methods of establishing weights are needed before weighting will really be useful.

In 1975, Einhorn and Hogarth [231] discussed a method of unit weighting where each variable was weighted by a +1.0 or a -1.0. They argued that by scaling the variables so that all zero-order correlations between the independent and dependent variables were positive, then the unit weighting method could be used for rank-order ordinal data.

V. Majority Rule-Graphical Methods

Certain majority-rule models emphasized graphical and spatial theory. These are summarized in the following paragraphs.

In 1968, Taylor [136] developed a combined graph-theoretical approach to aggregation of individual preference orders. The method used a "digraph" which is a directed graph which can be thought of as a set of points and some set of directional lines joining the points, which are the alternatives in the rankings. The digraph matrix, A, is analyzed by the Borda count, the Adjusted Borda count, and the Condorcet criterion. Then a second matrix, B, where $B = (b_{ij})$ when $b_{ij} = a_{ij} - a_{ji}$ if $a_{ij} > a_{ji}$, is used for a decision method. Then the row sums of the weighted sum of matrices is obtained through a power series type equation. The alternative ordering is given by the row sums of the sum of the series, if this sum exists. Taylor concludes that even if the digraph theory does not completely solve all aggregation problems, it makes the choice patterns more precise.

Wendell and Thorson [137] developed generalizations of social preference decisions using a majority rule model. The model utilized geometric symmetry and indifference contour techniques.

Good and Tideman [135] developed a method of collective ordering by assuming that the candidates can be located in "attribute space." The points lie on rays from the origin which form solid angles. The order of the absolute values of the angles determines the order of the points (alternatives). The approaches combine into one model a multidimensional scaling procedure, collective ordering, and a distribution of voter's ideal points in attribute space.

In 1976, Hoyer [94] utilized a spatial model of voting in his minimum loss method of determining aggregate preference orderings.

W. Resource Allocation By Voting

Some papers emphasized the decision for allocation of resources by a voting process followed by vote aggregation using majority rule techniques.

In 1970, Bernberg, Pondy, and Davis [138] listed the effect of the voting rule adopted by a capital budgeting committee on the allocation of resources. The rules tested were the Majority Rule, Unanimity Rule, and a Veto Rule. The results were matched to a model which was

weighted 67% as a gaming, voting stretegy and 33% random voting strategy. The results indicated that majority rule was the most efficient voting rule and it had a high degree of equity of personal payoff among group members.

In 1971, Pessemier and Baker [139] studied the application of three procedures for scaling the relative worth of R&D projects. First, simple ranked preference was used as a baseline to compare a cardinal dollar metric value method, a successive rating where each project is judged versus the most and least preferred projects, and a successive comparison method where each project is judged versus selected portfolios of projects. The methods were studied for two laboratories at different facilities in the same organization. The methods were evaluated within and between each lab. The Dollar Metric Value Method, essentially due to its added data content, was preferred. The study did not specifically compare the simple ranking procedure with the two methods.

In 1975, Souder [140] utilized a majority-rule method to achieve organizational consensus in specifying R&D project selection criteria at different organizations. The method was used with paired comparisons with group discussions, membership interactions; and re-evaluations of preferences. All pairs of criteria were judged in their permuted combinations, with a + scored where the column criterion is more important and a 0 otherwise. The criterion with the highest number of +'s is assigned a rank of 1, the next highest, 2, etc. The purpose of individual paired comparisons is to cause each subject to consider and document his perceived value structuring of the organizational goals.

X. Majority Rule-Minimum Distance Technique

Several authors explored theories and techniques to obtain a majority-rule consensus by optimizing a distance term between ranks.

In 1973, Bowman and Colantoni [141] developed a theory and methodology to constrain admissible majority rule rank-order decisions to those that will result in transitive aggregated order. The criterion for transitive selection is based upon minimizing a majority decision function. The authors show that the selection of a transitive group order is equivalent to an integer programming problem.

The majority rule, for aggregating weak, complete, and transitive individual orders has a proportion matrix P*:

$$P^*_{ij} = \frac{m_{ij} + 1/2\, m_{ij}^*}{m} \quad , \; i \neq j, \; [m_{ij}^* \text{ is m indifferent}]$$

$$P_{ii}^* = 1/2 \quad .$$

The majority rule matrix is based upon

$$\bar{P}_{ij} = 1 \text{ if } x_i P x_j,\ i \neq j$$

$$\bar{P}_{ij} = 0 \text{ if } x_j P x_i,\ i \neq j$$

$$\bar{P}_{ii} = 1/2 \quad .$$

The authors next define their majority decision function (mdf) to minimize, as

$$d(P^*, \bar{P}) = f[\Sigma_{ij} d_{ij}(P^*_{ij}, \bar{P}_{ij})] \quad .$$

Thus, they interpret the constrained d-majority decision problem as finding the minimum number of paired reversals that yield a transitive solution.

In 1974, Blin and Whinston [142] provided an alternate approach to the one of Bowman and Colantoni in characterizing majority voting theory through a quadratic assignment type problem. They suggest that this method may lead to more rapid development. One major strength claimed for the quadratic assignment formulation is that no explicit constraints are needed.

In 1978, Cook and Seiford [143] investigated the combining or ordinal preferences as priorities into a consensus. A concept of distance between rankings was theoreized, using the median ranking as a consensus. The distance measure is in the form $d(A, B) = \Sigma|a_i - b_i|$. This paper is one of few that recognize that the allocations of priorities to R&D projects is a frequent problem which utilizes the combining of individual preferences into a group consensus. The median ranking determination, with complete rankings, can be accomplished by one of several linear programming formulations, such as linear programming or an assignment problem. To guarantee transitivity, an integer programming formulation is necessary.

Y. Fuzzy-Set Rank Ordering

In recent years, researchers have explored the use of fuzzy-set theory as a method to better analyze the aggregation of individual preference rank orders. Several papers related to this paper are discussed in the following paragraphs.

In 1974, Blin, a noted author of social choice literature [144], proposed the concept that fuzzy (binary) relations as developed by Zadeh (1965) can be used for preference orderings of individuals and groups. An illustration is presented. The fuzzy relation would apply

as if x were strictly preferred to y, assign x a value of 1, and if y is strictly assigned to x, assign y a 0. All other non-strict pairs are assigned values between 0 and 1.

Blin [145] restated many of the same arguments given in [144]. In his conclusion, Blin added that the introduction of probability measures over fuzzy sets could be viewed as arising from a set of multiple observers' opinions over a repeated event.

In 1977, Jain [146] developed a method for decision making when alternatives are rated in terms of qualitative variables such as good, fair, very important, etc. Fuzzy sets are used to quantatively present the qualitative ratings. This paper presents only an outline of the decision method proposed.

In 1978, Orlovsky [147] continued the development of fuzzy set theory to be useful in preference relation. Fuzzy relation properties were defined for a fuzzy set which is a set of ordered pairs $[x, \mu_c(x)]$ where μ_c is a membership function. By use of a nondominated element method, rank orders were developed.

In 1978, Bezdek, Spillman, and Spillman [150] further developed fuzzy set theory for social order decisions. Relations and measures are developed to investigate fuzzy decision sets. An average fuzziness in R measure is F(R), which averages the joint preferences in R over all distinct pairs. F(R) is proportional to the fuzziness or uncertainty about pairwise rankings. A second measure, C(R) is the average certainty in R. C(R) averages the individual dominance of each distinct pair of rankings. C(R) is proportional to the overall certainty in matrix R. Bezdek, et al., show that $F(R) + C(R) = 1$ for a fuzzy set.

In 1979, Bezdek, Spillman, and Spillman [151] exemplified the application of the theory presented by Bezdek, et al. [150]. In this example, each individual was asked to determine his own fuzzy preference value (0, 1) for each pair of alternatives. These fuzzy values then became the a_{ij} entry for the individual's preference matrix. The aggregation of individual preferences, in this example by averaging, provided a fuzzy group preference matrix, R. The example continued through several time intervals until successive group R matrices stabilized.

In 1979, Watson, Weiss, and Donnell [149] presented a general discussion of fuzzy set decision analysis versus probabilistic decision analysis. Many decision makers should prefere a fuzzy methodology permitting them to describe their preferences by using the English language rather than always a number. Watson, et al. reported that Zadeh holds that imprecision and uncertainty are distinct qualities, which should be modeled with fuzzy set theory for imprecision and probability theory for uncertainty. Fuzzy set theory should be

considered as a parallel calculus to probability theory. In this paper, Watson, et al., showed how fuzzy set theory may be used to represent the imprecision of the probabilities and utilities of most decision analyses. The central concept of Zadeh's fuzzy set theory is the membership function which represents numerically (0→1) the degree to which an element belongs to a set.

The authors reiterated the characteristics of their methodology through several arguments. First, if imprecision is a different thought phenomenon, it should be implemented through fuzzy set theory. Second, there is no firm evidence as to preferred computational use. Third, two attractive aspects of the fuzzy approach are that fuzzy implications derive from interpreting set theory as a multivalued logic calculus and the fuzzy techniques are equivalent to multilevel interval analysis.

In 1979, Steinberg and Rinks [17] proposed a procedure for establishing a complete social preference ordering of a set of alternatives when group preference was fuzzy because of individual differences. The algorithm was applied to a civic project preference problem for a city. The first phase of the algorithm was to apply the Blin-Whinston (1974) procedure to obtain a linear ordering of the broad categories which contained multiple independent alternatives. The procedures included: obtain a linear ordering for alternatives with a category using the relation matrix, determine the degree of preference of adjoining pairs of the linear order, establish an anchor point to the least preferred in each category ordering, and map the ordering onto a line segment. Next information gathered from the preceding nonfuzzy relation is used to establish a preference order over all alternatives from the several categories.

The authors considered that the social preference function is a fuzzy relation and demonstrated a procedure to derive a nonfuzzy relation.

In 1979, Buckles [152] presented a literature survey of certain fuzzy set theory literature to describe methods to use fuzzy set theory for measuring the degree of group consensus and for providing an ordinal rank order for a list of preferences. The consensus method utilized the R, F(R), and G(R) method of Bezdek, et al. The rank order method reflected the procedure of Orlovsky, but Buckles interpreted the complex notation into an understandable and usable procedure.

Whaley [232] presented a scenario-type paper to explain the use of a Fuzzy Decision Making (FDM) computer program that was written in BASIC language and was interactive. A copy of the program listing is appended to the paper. FDM has a 0-1 scale for the decision maker to use to determine cost in terms of needs and utility value. The program combines these judgments to produce relative scale values. The paper does not derive the mathematical basis for the program but does list its references for researching the technique.

IV. SUMMARY

This report has presented an overview of the literature that has been identified as applicable to the theory and practical methods used to aggregate multiple lists of prioritized R&D projects product requirements. The relevant social choice theory is extensive and diverse. Majority-rule methods are described and compared. A few practical applications are discussed.

This literature, together with the dissertation, Dobbins [1] and the computer program and procedure report, Dobbins [233] summarize the research done to date and suggest areas for further investigation.

The following tables are provided as a cross reference for the literature in the references. The surnames of the authors listed first for each book and periodical are listed in alphabetical order. They are followed by their respective item numbers in the references. The topic group letter identity is given for each item.

BOOKS

First Author's Surname	Reference Number	Topic Area
Arrow	26	F
Arrow	56	I
Aumann	187	H
Black	8	A
Buchanan	111	Q
Chamberlain	163	C
Cochrane	101	O
Cohon	102	O
Conover	158	B
Coombs	97	D
Crook	192	J
Dobbins	1	K
Dobbins	233	K
Edwards	156	B
Edwards	157	B
Fishburn	17	D,H
Haith	198	K
Hollander	160	B
Hanson	114	Q
Hoyer	94	N,V
Kendall	5	B
Kirkwood	162	C
Lazerfield	214	O
Lehmann	159	B
Luce	50	H
Pattanaik	51	H
Pattanaik	52	H
Robbins	3	A
Rothenberg	103	O
Seaver	215	O
Sen	100	O
Sheple	230	T
Shisko	106	P
Seigel	155	B
Singamsetti	223	T
Sticha	217	O
Svestka	78	K
Thrall	197	K
Tullock	112	Q
Van den Bogaard	93	N
Von Neumann	186	H
Wendell	137	V
Wyatt	79	K

PERIODICALS

First Author's Surname	Reference Number	Topic Area
Abrams	119	S
Allen	73	J
Armstrong	18	D
Arrow	27	F
Arrow	113	Q
Bacharach	180	G
Barbut	105	P
Barton	115	Q
Bartoszynski	166	D
Bell	23	E
Bell	176	E
Bergson	4	A
Bezdek	150	Y
Bezdek	151	Y
Birnberg	138	W
Black	9	C
Black	10	E
Black	161	C
Black	196	J
Blair	183	G
Blau	39	G
Blau	128	T
Blin	188	H
Blin	11	C,G
Blin	142	X
Blin	144	Y
Blin	145	Y
Blin	170	D
Blin	185	G
Bowman	141	X
Bowman	195	J
Bowman	204	L
Brown	104	O
Buchanan	29	F
Buckles	152	Y
Buckley	107	P
Buckley	218	P
Campbell	41	G
Campbell	116	R
Campbell	173	D
Campbell	221	S
Castore	81	K
Chernous'ko	131	T

PERIODICALS (CONTINUED)

First Author's Surname	Reference Number	Topic Area
Cole	212	M
Colman	74	J
Cook	143	X
Cook	168	D
Dalkey	179	G
Davis	31	F
Davis	200	L
De Grazia	2	A
De Meyer	57	I
Denzau	12	C
Dummett	13	C
Dutta	24	E
Ehrenberg	154	B
Einhorn	231	U
Farris	33	F
Feldman	44	G
Ferejohn	34	F
Ferejohn	43	G
Ferejohn	55	H
Fine	127	D,T
Fishburn	20	E
Fishburn	21	D
Fishburn	22	E
Fishburn	45	G
Fishburn	58	I
Fishburn	64	J
Fishburn	65	J
Fishburn	75	J
Fishburn	83	L
Fishburn	88	M
Fishburn	89	M
Fishburn	90	M
Fishburn	169	D
Fishburn	191	L
Fishburn	193	J
Fishburn	194	J
Fishburn	211	M
Fishburn	213	M
Friedman	6	B
Garman	120	J,S
Gehrlein	121	S
Gevers	59	I
Gibbard	110	P

PERIODICALS (CONTINUED)

First Author's Surname	Reference Number	Topic Area
Gillett	71	J
Good	135	V
Goodman	38	G
Grandmont	91	M
Grandmont	174	D,L
Greenberg	25	E
Gustafson	133	U
Hamada	167	D
Hansson	46	G
Harnett	15	D
Herzberger	54	H
Hildreth	47	G
Hinich	222	T
Hoyer	95	N
Inada	40	G,L,M
Inada	48	G,M
Inada	202	L,M
Jain	146	Y
Jamison	84	L
Jamison	205	L
Kaneko	208	L
Keeney	62	I
Kelly	226	T
Kemp	190	I
Kendall	7	B
Klahr	122	S
Klee	132	U
Koopmans	92	M
Kramer	207	L
Kuga	42	G
Little	28	F
Luce	87	M
May	37	G
May	82	L
McGuire	35	F
McKelvey	210	L,P
McKelvey	227	T
Merchant	209	L
Monjardet	184	G
Moon	76	J
Murakami	32	F
Murakami	125	T
Moran	153	B

PERIODICALS (CONTINUED)

First Author's Surname	Reference Number	Topic Area
Nakamura	108	P
Navarrete	175	D
Niemi	123	S
Nitzan	225	T
Orlovsky	147	Y
Ostojic	30	F
Paris	72	J
Parks	182	T
Pattanaik	126	T
Peleg	60	I
Pessemier	139	W
Plott	216	O
Pomeranz	203	L
Rader	228	T
Rao	206	L
Riker	98	O
Richelson	68	J
Richelson	69	J
Richelson	70	J
Richelson	129	T
Rowse	134	U
Rothenberg	189	I
Rubinstein	219	P
Salles	109	G,P
Samuelson	63	I
Saposnik	85	L
Schuyler	14	D
Schwartz	201	L
Sen	86	L
Sen	177	G
Sen	178	G
Sen	201	L
Sengupta	229	T
Shannon	16	D
Shannon	117	R
Shepsle	230	T
Sheridan	172	D
Smith	17	D
Souder	140	W
Steinberg	148	Y
Straffin	165	D
Taylor	136	V
Taylor	164	D

PERIODICALS (CONCLUDED)

First Author's Surname	Reference Number	Topic Area
Taylor	165	D
Theil	96	N
Tullock	220	Q
Ward	118	R
Watson	149	Y
Weisberg	124	S
Wendell	224	F
Whaley	232	Y
Wilson	49	G
Wilson	181	G
Winkler	130	U
Wood	19	D
Young	61	I
Young	66	J
Young	67	J
Young	77	J
Young	199	K

REFERENCES

1. Dobbins, Edward B., Jr., A Methodology For Aggregation Of Multiple Criteria Rank-Ordered Priorities, Huntsville, AL, University of Alabama In Huntsville, AL, May 1980.*

2. De Grazia, Alfred, "Mathematical Derivation of an Election System," Isis, Vol. 44, June 1953, pp. 42-51. A,J

3. Robbins, L., An Essay On The Nature Significance of Economic Science, London: MacMillan Press, 1935, 1972. A

4. Bergson, A., "A Reformulation of Certain Aspects of Welfare Economics," Quarterly Journal of Economics, Vol. 52, 1938, pp. 310-334. A

5. Kendall, M. G., Rank Correlation Methods, New York: Hafner Publishirg 1962. B

6. Friedman, M., "A Comparison of Alternative Tests of Significance For The Problem of M Rankings," Annals of Math. Statistics, Vol. II, 1940, p.86+. B

7. Kendall, M. G. and Smith, B. B., "The Problem of M Rankings," Annals of Math. Statistics, Vol. X, 1939, pp. 275-287. B

8. Black, D., Committees and Elections, Cambridge, Great Britian: University Press, 1958. A

9. Black, Duncan, "On the Rationale of Group Decision Making," Jul of Political Economy, Vol. 36, February 1948, pp. 23-34. C

10. Black, D., "The Elasticity of Committee Decisions With An Altering Size of Majority," Econometrica, Vol. 16, 1948, pp. 262-270. E

11. Blin, J. M. and Satterthwaits, M. A., "Strategy-Proofness and Single-Peakedness," Public Choice, Vol. 26, Summer 1976, pp. 53-58. C,G

12. Denzau, A. T., Parks, R. P., "Continuity of Majority-Rule Equilibrium," Econometrica, Vol. 43, 1975, pp. 858-866. C

13. Dummett, Michael and Farquharson, Robin, "Stability in Voting," Econometrica, Vol. 29, January 1961, pp. 33-43. C

14. Schuyler, G. L., "The Ordering of n Items Assigned to k Rank Categories by Votes of m Individuals," American Statistical Association Journal, Vol. 43, December 1948, pp. 559-563. D

15. Harnett, D. L., "A Level of Aspiration Model for Group Decision Making," Journal of Personality and Social Psychology, Vol. 5, 1967, pp. 58-66. D

*Also published as Management Report RN-80-3, US Army Missile Command, Redstone Arsenal, Alabama, May 1980.

16. Shannon, R. E., "The Amalgamation Of Group Opinions Or Evaluations," Journal of Industrial Engineering, Vol XIX, No. 6, June 1968, pp. xviii. D

17. Smith, J. H., "Aggregation of Preferences and Variable Electorate," Econometrica, Vol. 41, 1973, pp. 1027-1041. D

18. Armstrong, R. D., Cook, W. D., "Strategy Selection in Ordinal Ranking Problems, "Research Report CCS 299, Univ. of Texas, Austin, July 1977. D

19. Wood, R., and Wilson, D. T., "Technique For Converting Ranks Into Measures," Brit. Journal of Psych., Vol. 68, August 1977, pp. 321-326. D

20. Fishburn, P. C., "Voter Concordance, Simple Majorities, and Group Decision Methods," Behavioral Science, Vol. 18, 1973, pp. 364-376. E

21. Fishburn, P. C., "Single-Peaked Preferences and Probabilities of Cylical Majorities," Behavioral Sciences, Vol. 19, 1974, pp. 21-27. D

22. Fishburn P. C., Gehrlin, W. V., "Win Probabilities and Simple Majorities in Probabilistic Voting Situations," Mathematical Programming, Vol. 11, No. 1, 1976, pp. 28-41. E

23. Bell, Colin E., "What Happens When Majority Rule Breaks Down?" Public Choice, Vol. 33, Issue 2, 1978, pp. 121-126. E

24. Dutta, Bhaskar and Pattanaik, Prasanta, K., "On Nicely Consistent Voting Systems," Econometrica, Vol. 46, No. 1, January 1978, pp. 163-170. E

25. Greenberg, Joseph, "Consistent Majority Rules Over Compact Sets of Alternatives," Econometrica, Vol. 47, No. 3, May 1979, pp. 627-636. E

26. Arrow, K. J., Social Choice and Individual Values, New Haven, London: John Wiley, 1963. F

27. Arrow, K. J., "Rational Choice Functions and Orderings," Economica, Vol. 26, pp. 121-127, 1959. F

28. Little, I. M. D., "Social Choice and Individual Values," J. of Pol. Econ., Vol. 60, 1952, pp. 422-432. F

29. Buchanan, J. M., "Social Choice, Democracy, and Free Markets," Journal of Political Economy, Vol. 62, April 1954, pp. 114-123. F

30. Ostojic, S., "Modern Trends in Cybernetics," Third Int. Cong. of Cybernetics, Vols. 1, 2, 3, 1975, pp. 575-581. F

31. Davis, Otto, A., et al, "Social Preference Orderings and Majority Rule," Econometrica, Vol. 40, January 1972, pp. 147-157. F

32. Murakami, Y., "Some Logical Properties of Arrowian Social Welfare Function," The Journal of Economic Behavior, Vol. 1, No. 1, April 1961, pp. 77-84. F

33. Farris, D. R. and Sage, A. P., "Introduction and Survey of Group Decision Making With Application to Worth Assessment," IEEE Transactions on Systems, Man and Aybernetics, Vol. SMC-5, No. 3, May 1975. F

34. Ferejohn, J. A., "Decisive Coalitions in the Theory of Social Choice," Journal of Econ. Theory, Vol. 15, August 1977, pp. 301-306. F

35. McGuire, J. B. and Thompson, "On the Reconstruction of an Evolutionary Order," J. Theor. Bio., Vol. 75, 1978, pp. 141-147. F

36. Schwartz, T., "Rationality And The Myth Of The Maximum," Nous, Vol. 6, 1972, pp. 97-117. F

37. May, K. O., "A Set of Independent Necessary and Sufficient Conditions for Simple Majority Decision," Econometrica, Vol. 20, 1952, pp. 680-684. G

38. Goodman, L. A. and Markowitz, H., "Social Welfare Functions Based On Individual Rankings," American Journal of Sociology, Vol. 58, 1952, pp. 257-262. G

39. Blau, J. H., "The Existence of Social Welfare Functions," Econometrica, Vol. 25, January 1957, pp. 302-313. G

40. Inada, K., "A Note on the Simple Majority Decision Rule," Econometrica, Vol. 32, October 1964, pp. 525-531. L,M

41. Campbell, D. E., "Social Choice and Intensity of Preference," Journal of Political Economy, Vol. 81, 1973, pp. 211-218. G

42. Kuga, K. and Nagatani, H., "Voter Antagonism and The Paradox of Voting," Econometrica, Vol. 42, No. 6, November 1974, pp. 1045-1067. G

43. Ferejohn, J. A. and Grether D. M., "On a Class of Rational Social Decision Procedures," Journ. of Econ. Theory, Vol. 8, 1974, pp. 471-482. G

44. Feldman, A. M., "Very Unsubtle Version of Arrow's Impossibility Theorem," Econ. Inquiry, Vol. 12, December 1974, pp. 534-546. G

45. Fishburn, P. C., "Arrow's Impossibility Theorem: Concise Proof and Infinite Voters," J. Econ. Theory, Vol. 2, 1970, pp. 103-106. G

46. Hansson, B., "Group Preferences," Econometrica, Vol. 37, 1969, pp. 50-54. G

47. Hildreth, C., "Alternative Conditions for Social Orderings," Econometrica, Vol. 21, 1953, pp. 81-94. G

48. Inada, Ken-Ichi, "On the Economic Welfare Function," Econometrica, Vol. 32, 1964, pp. 316-338. G,M

49. Wilson, R. B., "Social Choice Theory Without Pareto Principle," Journal of Econ. Theory, Vol. 5, 1972, pp. 478-486. G

50. Luce, D. R. and Raiffa, H., Games and Decisions, New York: John Wiley, 1957. H

51. Pattanaik, P. K., Voting and Collective Choice Some Aspects of the Theory of Group Decision-Making, Cambridge University, 1971. H

52. Pattanaik, P. K., Strategy and Group Choice, New York: North Holland Publishing, 1978. H

53. Fishburn, P. C., The Theory of Social Choice, Princeton, New Jersey: Princeton University Press, 1973. H,D

54. Herzberger, H. G., "Ordinal Preference and Rational Choice," Econometrica, Vol. 41, 1973, pp. 187-237. H

55. Ferejohn, J. and Page, T., "On the Foundations of Intertemporal Choice," Am. J. Agric. Econ., May 1978, pp. 269-275. H

56. Arrow, K. J., Selected Readings in Economic Theory From Econometrica, Cambridge, MA: MIT Press, pp. 302-313. I

57. DeMeyer, F. and Plott, "A Social Welfare Function Using 'Relative Intensity' of Preference," Quarterly Journal of Economics, Vol. 85, 1971, pp. 179-186. I

58. Fishburn, P. C., "Comments on Hansson's Group Preferences," Econometrica, Vol. 38, 1970, pp. 184-185. I

59. Gevers, Louis, "On Interpersonal Comparability and Social Welfare Orderings," Econometrica, Vol. 47, No. 1, January 1979, pp. 75-89. I

60. Peleg, B., "Consistent Voting Systems," Econometrica, Vol. 46, 1978, p. 153. I

61. Young, H. P., "A Note On Preference Aggregation," Econometrica, Vol. 42, 1974, pp. 1129-1131. I

62. Keeney, R. L., "Group Preference Axiomatization with Cardinal Utility," Management Science, Vol. 23, October 1976, pp. 140-145. I

63. Samuelson, P., "Reaffirming The Existence of 'Reasonable' Bergson-Samuelson Social Welfare Functions," Economica, Vol. 44, February 1977, pp. 81-88. I

64. Fishburn, P. C., "A Comparative Analysis of Group Decision Methods," Behavioral Science, Vol. 16, 1971, pp. 538-544. J

65. Fishburn, P. C., and Gehrlin, W. V., "Borda's Rule Positorial Voting, and Condorcet's Simple Majority Principle," Public Choice, Vol. XXVIII, Winter 1976, pp. 79-88. J

66. Young, H. P., "An Axiomatization of Borda's Rule," Journal of Econ. Theory, Vol. 9 , 1974, pp. 43-52. J

67. Young, H. P., "Social Choice Scoring Functions," SIAM J. Appl. Math., Vol. 28, June 1975, pp. 824-834. J

68. Richelson, J. T., "A Comparative Analysis of Social Choice Functions," Behavioral Science, Vol. 20, 1975, pp. 331-342. J

69. Richelson, J. T., "A Comparative Analysis of Social Choice Functions, II," Behavioral Science, Vol. 23, 1978, pp. 38-44. J

70. Richelson, J. T., "A Comparative Analysis of Social Choice Functions, III," Behavioral Science, Vol. 23, 1978, pp. 169-176. J

71. Gillett, Raphael, "Collective Indecision," Behavioral Science, Vol. 22, 1977, pp. 383-390. J

72. Paris, D. C., "Plurality Distortion and Majority Rule," Behavioral Science, Vol. 20, 1975, pp. 125-135. J

73. Allen, Glen O., "Beyond the Voter's Paradox," Ethics, Vol. 88, October 1977, pp. 50-61. J

74. Colman, A. M. and Pountney, I., "Borda's Voting Paradox: Theoretical Likelihood and Electoral Occurrences," Behavioral Science, Vol. 23, 1978, pp. 15-20. J

75. Fishburn, P. C. and Gehrlein, W. V., "Analysis of Simple Two Stage Voting Systems," Behavioral Science, Vol. 21, 1976, pp. 1-21. J

76. Moon, J. W., "A Problem on Rankings by Committees," Econometrica, Vol. 44, No. 2, March 1976, pp. 241-246. J

77. Young, H. P. and Levenglick. "A Consistent Extension of Condorcet's Election Principle," SIAM J. of Appl. Math., Vol. 35, 1978, pp. 285-300. J

78. Svestka, J. A., The Development of a Rational Group Ordering Technique Without the Use of Cardinal Utilities, A Thesis, University of Alabama in Huntsville, Huntsville, Alabama, 1966. K

79. Wyatt, M. W., Amalgamating the Rankings of Judges With Various Degrees of Experties, A Thesis, University of Alabama in Huntsville, Huntsville, Alabama, 1973.

80. Chartier, O. and Wertheimer, S., "Voting Simulation," 5th Annual Pittsburgh Conference on Modeling and Simulation, April 1974, pp. 24-26. K

81. Castore, Carl H., Peterson, Kevin, Goodrich, Thomas A., "Risky Shift: Social Value or Social Choice? An Alternative Model," Journal of Personality and Social Psychology, Vol. 30(3), December 1971, pp. 487-494. K

82. May, K. O., "Intransitivity, Utility, and The Aggregation of Preference Patterns," Econometrica, Vol. 22, 1954, pp. 1-13. L

83. Fishburn, P. C., "Intransitive Individual Indifference and Transitive Majorities," Econometrica, Vol. 38, No. 3, May 1970, pp. 482-489. L

84. Jamison, D., "The Probability of Intransitive Majority Rule: An Empirical Study," Public Choice, Vol. 23, 1975, pp. 87-94. L

85. Saposnik, R., "On Transitivity of Social Preference Relation Under Simple Majority Rule," Journal of Economic Theory, Vol. 10, No. 1, 1975, pp. 1-7. L

86. Sen, A. K., "Choice Functions and Revealed Preference," Review of Economic Studies, Vol. 38, 1971, pp. 307-317. L

87. Luce, R. D., "Semiorders and a Theory of Utility Discrimination," Econometrica, Vol. 24, No. 2, April 1956, pp. 272-285. M

88. Fishburn, P. C., "On the Family of Linear Extensions of a Partial Order," J. Comb. Theory B (USA), Vol. 17, No. 3, 1974, pp. 240-3. M

89. Fishburn, P. C., "Impossibility Theorems Without the Social Completeness Axiom," Econometrica, Vol. 42, No. 4, July 1974, pp. 695-704. M

90. Fishburn, P. C., "Multicriteria Choice Functions Based on Binary Relations," Operations Research, Vol. 25, 1977, pp. 989-1012. M

91. Grandmont, J. M., "Intermediate Preferences and the Majority Rule," Econometrica, Vol. 46, No. 2, 1978, pp. 317-330. M

92. Koopman, T. C., "On Flexibility of Future Preference," Human Judgments and Optimality, 1964, 243-254. M

93. Van den Bogaard, P. J. M. and Versluis, J., The Design of Socially Optimal Decision, New York: John Wiley, 1960. N

94. Hoyer, R. W., Some Multivariate Problems of a Spatial Model of Voting Under Majority Rule, a Virginia Polytechnic Institute Dissertation, 1976. N,V

95. Hoyer, R. W. and Mayer, L., "Social Preference Orderings Under Majority Rule," Econometrica, Vol. 43, July 1975, pp. 803-806. N

96. Theil, H., "On The Symmetry Approach To The Committee Decision Problem," Management Science, Vol. 9, 1963, pp. 380-393. N

97. Coombs, C. H., A Theory of Data, New York: John Wiley, 1964. O

98. Riker, W., "Voting and The Summation of Preference: An Interpretative Bibliographical Review of Selected Developments During the Last Decase," Amer. Political Sci. Rev., Vol. 55, 1961, pp. 900-end. O

99. Sen, A. K., Collective Choice and Social Welfare, San Francisco: Holden-Day Inc., 1970. O

100. Sen, A. K., Collective Choice and Social Welfare, San Francisco: Holden-Day Inc., 1970. O

101. Cochrane, J. L. and Zeleny, M., Multiple Criteria Decision Making, University of South Carolina, 1973. O

102. Cohon, J. L., Multiobjective Programming and Planning, New York: Academic Press, 1978. O

103. Rothenberg, J., The Measurement of Social Welfare, Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1961. O

104. Brown, D. J., "Aggregation of Preferences," Quarterly Journal of Econ. Theory, Vol. 89, No. 3, August 1975, pp. 456-469. O

105. Barbut, Marc, "Does the Majority Ever Rule?" Portfolio and Art New Annual, Vol. 4, 1961, pp. 16-83, 161-168. P

106. Shisko R., A Survey of Solution Concepts for Majority Rule Games, Santa Monica: The Rand Corporation, 1974. P

107. Buckley, J. J. and Westen, T. E., "The Majority Rule Game," Int. J. Game Theory, Vol. 4, No. 1-2, 1975, pp. 105-112. P

108. Nakamura, K., "The Core of a Simple Game With Ordinal Preferences," Int. Journal of Game Theory, Vol. 4, Issue 2, 1975, p. 95. P

109. Salles, M., Wendell, R. E., "A Further Result On The Core Of Voting Games," Int. J. Game Theory, Vol. 6, No. 1, 1977, pp. 35-40. G,P

110. Gibbard, Allan, "Manipulation of Schemes That Mix Voting With Chance," Econometrica, Vol. 45, No. 3, April 1977, pp. 665-681. P

111. Buchanan, J. and Tullock, G., The Calculus of Consent, Ann Arbor: University of Michigan, 1962. Q

112. Tullock, C., Toward a Mathematic of Politics, Ann Arbor: University of Michigan Press, 1967. Q

113. Arrow, K. J., "Tullock and an Existence Theorem," Public Choice, Vol. 6, 1969, p. 105. Q

114. Hanson, R. A., Majority Rule and Policy Outcomes: A Critique of the "Calculus of Consent," University of Minnesota Dissertation, 1973. Q

115. Barton, David M., "Constitutional Choice and Simple Majority Rule: Comment," Journal of Political Economy, Vol. 81, 1973, pp. 471-480. Q

116. Campbell, Donald E., "Income Distribution Under Majority Rule and Alternative Taxation Criteria," Public Choice, Vol. 22, Summer 1975, pp. 23-35. R

117. Shannon, R. E. and Biles, W. E., "The Utility of Certain Curriculum Topics To Operations Research Practitioners," Operations Research, Vol. 18, No. 4, July-August 1970, pp. 741-745. R

118. Ward, B., "Majority Voting and Alternative Forms of Public Enterprise," Second Conference In Urban Expenditure, 1963. R

119. Abrams, R., "The Voter's Paradox and the Homogeniety of Individual Preference Orders," Public Choice, Vol. 26, Summer 1976, pp. 19-27. S

120. Garman, Mark B., Kamien, Morton, I., "The Paradox of Voting: Probability Calculations," Behavioral Science, Vol. 13(4), 1968, pp. 306-316. J,S

121. Gehrlein, William V., Fishburn, P. C., "Condorcet's Paradox and Anonymous Preference Profiles," Public Choice, Vol. 26, Summer 1976, pp. 1-18. S

122. Klahr, D., "A Computer Simulation of the Paradox of Voting," American Political Science Review, Vol. 60, 1966, pp. 384-390. S

123. Niemi, R. G. and Weisberg, H. F., "A Mathematical Solution for the Probability of the Paradox of Voting," Behavioral Science, Vol. 13, 1968, p. 317. S

124. Weisberg, H. F. and Niemi, R. G., "A Pairwise Probability Approach To The Likelihood Of The Paradox Of Voting," Behavioral Science, Vol. 18, 1973, pp. 109-117. S

125. Murakami, Y., "Formal Structure of Majority Decision, Econometrica, Vol. 34, 1966, pp. 709-718. T

126. Pattanaik, P. K., "Sufficient Conditions for the Existence of a Choice Set Under Majority Voting," Econometrica, Vol. 38, No. 1, January 1970, pp. 165-170. T

127. Fine, Kit, "Some Necessary and Sufficient Conditions for Representative Decision on Two Alternatives," Econometrica, Vol. 40, 1972, pp. 1083-1090. T,D

128. Blau, J. H. and Deb, Rajat, "Social Decision Functions and the Veto," Econometrica, Vol. 45, No. 4, May 1977, pp. 871-879. T

129. Richelson, J., "Conditions on Social Choice Functions," Public Choice, Vol. XXXI, Fall 1977, p. 79. T

130. Winkler, R. L., "The Consensus of Subjective Probability Distributions," Management Science, Vol. 15, No. 2, October 1968, pp. B61-B75. U

131. Chernous'ko, F. L., "Weight Factors in Expert Estimates," Cybernetics, No. 6, October 1972, pp. 1021-1024. U

132. Klee, A. J., "The Utilization of Expert Opinion," Amer. Inst. of Chemical Eng., Vol. 18, November 1972, pp. 1107-1115. U

133. Gustafson, D. H., Pai, G. K. and Cramer, G. C., "A Weighted Aggregate Approach to R&D Project Selection," AIIE Transactions, Vol. 3, March 1971, pp. 22-31. U

134. Rowse, G., Gustafson, D., Ludke, R., "Comparison of Rules For Aggregating Subjective Likelihood Ratios," Organizational Behavior and Human Perf., Vol. 12, 1974, pp. 274-285. U

135. Good, I. J., Tideman, T. N., "From Individual to Collective Ordering Through Multidimensional Attribute Space," Proc. R. Soc. A (BG), Vol. 347, No. 1650, 1975, pp. 371-385. V

136. Taylor, M. J., "Graph-Theoretical Approaches To Theory Of Social Choice," Public Choice, Vol. 4, 1968, pp. 35-47. V

137. Wendell, R. E., Thorson, S. J., "Some Generalizations of Social Decisions Under Majority Rule," Econometrica, Vol. 42, No. 5, 1974, pp. 893-912. V

138. Bernberg, J. G., Pondy, L. R., Davis, C. L., "Effect of Three Voting Rules on Resource Allocation Decisions," Management Science, Vol. 16, No. 6, February 1970, pp. 356-372. W

139. Pessemier, E. A. and Baker, N. R., "Project and Program Decisions In R&D," R&D Management, Vol. 2, October 1971, pp. 3-14. W

140. Souder, W. E., "Achieving Organizational Censensus With Respect To R&D Project Selection Criteria," Managment Science, Vol. 21, 1975, pp. 669-681. W

141. Bowman, V. J. and Colantoni, C. S., "Majority Rule Under Transitivity Constraints," Management Science, Vol. 19, May 1973, pp. 1029-1041. X

142. Blin, J. M. and Whinston, A. B., "A Note on Majority Rule Under Transitivity Constraints," Management Science, Vol. 20, No. 11 July 1974, pp. 1439-1440. X

143. Cook, W. D. and Seiford, L. M., "Priority Ranking and Concensus Formation," Management Science, Vol. 24, No. 16, December 1978, p. 1732. X

144. Blin, J. M, "Fuzzy Relations in Group Decision Theory," Journal of Cybernetics, Vol. 4, No. 2, April-June 1974, pp. 17-22. Y

145. Blin, J. M., and Whinston, A. B., "Fuzzy Sets and Social Choice," Journal of Cybernetics, Vol. 3, No. 4, 1974, pp. 28-36. Y

146. Jain, R., "A Procedure for Multiple-Aspect Decision Making Using Fuzzy Sets," Int. J. Systems Sci., Vol. 8, No. 1, 1977, pp. 1-7. Y

147. Orlovsky, S. A., "Decision-Making With A Fuzzy Preference Relation," Fuzzy Sets and Systems 1, No. 3, 1978, pp. 155-167. Y

148. Steinberg, Earle and Rinks, Dan, "An Application Of The Blin-Whinston Algorithm Resolving Fuzzy Group Preferences," Management Science, Vol. 25, No. 2, February 1979, pp. 159-170. Y

149. Watson, S. R., Weiss, J. J., and Donnell, M. L., "Fuzzy Decision Analysis," IEEE Transactions on Systems, Man and Cybernetics, Vol. SMC-9, No. 1, January 1979, pp. 1-9. Y

150. Bezek, James C. et al., "A Fuzzy Relations Space for Group Decision Theory," Fuzzy Sets and Systems 1, No. 4, October 1978, pp. 255-268. Y

151. Bezek, J. C., et al, "Fuzzy Relation Spaces for Group Decision Theory: An Application," Fuzzy Sets and Systems 2, No. 1, January 1979, pp. 5-14. Y

152. Buckles, B. P., "Two Applications of Fuzzy Set Theory to Decision Making," Literature Survey, Summer 1971. Y

153. Moran, P. A. P., "On The Method of Paired Comparisions," Biometrika, Vol. 34, 1947, pp. 363-365. B

154. Ehrenberg, A. S. C., "On Sampling From a Population of Rankers," Biometrica, Vol. 39, 1952, pp. 82-87. B

155. Siegel, S., Nonparametric Statistics for Behaviorial Sciences, New York: McGraw-Hill, 1956. B

156. Edwards, A. L., Techniques of Attitude Scale Conservations, New York: Appleton Century Crafts, 1957. B

157. Edwards, Allen L., Statistical Methods for the Behavioral Sciences, Chapter 19, New York: Holt, Rinehart, 1963. B

158. Conover, W. J., Practical Nonparametric Statistics, New York: John Wiley, 1971. B

159. Lehmann, E. L. and D'Abera, H., Nonparametrics: Statistical Methods Based on Ranks, San Francisco, California: Holden Day, Inc., 1975. B

160. Hollander, M. and Sethuraman, J., Testing for Agreement Between Two Groups of Judges, Florida State University, 1977. B

161. Black, Duncan, "The Decisions of a Committee Using a Special Majority," Econometrica, Vol. 16, January 1948, pp. 245-261. C

162. Kirkwood, C. W., Decision Analysis Incorporating Decisions of Groups, Massachusetts Institute of Technology, 1972. C

163. Chamberlain, J. R., Essays on Public Goods, Externalities and Majority Rule, Stanford University Dissertation, 1974. C

164. Taylor, M., "Proof Of A Theorem On Majority Rule," Behavioral Science, Vol. 14(3), 1969, pp. 228-231. D

165. Taylor, M., "Mathematical Political Theory," British Journal Of Political Science, Vol. 1, No. 3, July 1971, p. 340. D

166. Bartoszynski, Robert, "Power Structure in Dichotomous Voting," Econometrica, Vol. 40, November 1972, pp. 1003-1019. D

167. Hamada, K., "Simple Majority Rule on Distribution of Income," Journal of Economic Theory, Vol. 6, No. 3, 1973, pp. 243-264. D

168. Cook, W. D. and Saipe, A. L., "Committee Approach to Priortiy Planning: The Median Ranking Method," Cahiers du Centre d'Etude de Rocherch Operationnelle, Vol. 8(3), 1973, pp. 337-350. D

169. Fishburn, P. C., "Three-Valued Representative Systems," Mathematical Systems Theory, Vol. 9, No. 3, 1975, pp. 265-280. D

170. Blin, J. M. and Whinston, A. B., "Discriminant Functions and Majority Voting," Management Science, Vol. 21, January 1975, pp. 557-566. D

171. Straffin, P. D., "Majority Rule and General Decision Rules," Theory and Decision, Vol. 8, No. 4, 1977, pp. 351-360. D

172. Sheridan, T. B. and Sicherman, A., "Estimation Of A Groups's Multiattribute Utility Function In Real Time By Anonymous Voting," IEEE Transactions On Systems, Man and Cybernetics, Vol. SMC-7, No. 5, May 1977, pp. 392-394. D

173. Campbell, Donald E., "Realization of Choice Functions," Econometrica, Vol. 46, 1978, p. 171. D

174. Grandmont, J. M., "Intermediate Preferences and the Majority Rule," Econometrica, Vol. 46, No. 2, 1978, pp. 317-330. D,L

175. Navarrete, N. Jr., Fukushima, M., and Mine, H., "A New Ranking Method Based on Relative Position Estimate and Its Extensions," IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-9, No. 11, November 1979, pp. 681-689. D

176. Bell, Colin E., "What Happens When Majority Rule Breaks Down?," Public Choice, Vol. 33, Issue 2, 1978, pp. 121-126. E

177. Sen, A. K., "A Possibility Theorem On Majority Decisions," Econometrica, Vol. 34, 1966, pp. 491-499. G

178. Sen, A. K., "Interpersonal Aggregation and Partial Comparability," Econometrica, Vol. 38, No. 3, May 1970, pp. 393-409. G

179. Dalkey, N., "An Impossibility Theorem for Group Probability Functions," Presented at NSF Conf. on Subjective Optimality, July 16-21, 1972, Rand P-4862. G

180. Bacharach, Michael, "Group Decisions in the Face of Differences of Opinion," Management Science, Vol. 22, October 1975, pp. 182-191. G

181. Wilson, R. B., "On The Theory of Aggregation," Journal Of Econ. Theory, Vol. 10, 1975, pp. 89-99. G

182. Parks, R. P., "Impossibility Theorem For Fixed Preferences," Rev. Economic Studies, Vol 43, October 1976, pp. 447-450. T

183. Blair, D. H., et al., "Impossibility Theorems Without Collective Rationality," Journal of Economic Theory, Vol. 13, December 1976, pp. 361-379. G

184. Monjardet, B., "UneAutre Preuve Du Theoreme D'Arrow," Recherche Operationnelle, Vol. 12, No. 3, 1978, pp. 291-296. G

185. Blin, J. M. and Satterthwaite, M. A., "Individual Decisions and Group Decisions," Journal of Public Economics, Vol. 10, No. 2, 1978, pp. 247-267. G

186. Von Neumann, J. and Morgenstern, O., Theory of Games and Economic Behavior, Princeton, New Jersey: Princeton University Press, 1953. H

187. Aumann, Robert J., Human Judgments and Optimality, Chapter 12, New York: John Wiley, 1964. H

188. Blin, J. M, et al., "Optimization Theory and Social Choice," Proc. of 6th Hawaiian Int. Conf. on Sys. Sci. Suppl. on Urban and Regional Systems: Modelling Analysis and Decision Making, 1973, pp. 158-161. H X

189. Rothenberg, Jerome, "Conditions For A Social Welfare Function," J. of Pol. Econ., Vol. 61, 1953, pp. 389-405. I

190. Kemp, M. C. and Ng, Y. K., "On Existence of Social Welfare Functions, Social Ordering, and Social Decision Functions," Economica, Vol. 43, February 1976, pp. 59-66. I

191. Fishburn, P. C., "The Irrationality of Transitivity in Social Choice," Behavioral Science, Vol. 15, 1970, pp. 119-123. L

192. Crook, J. F., Separate Versus Concurrent Decision Procedures for Public Goods Provision Under Majority Rule, Virginia Polytechnic Institute Dissertation, 1973. J

193. Fishburn, P. C., "Simple Voting Systems and Majority Rule," Behavioral Sciences, Vol. 19, 1974, pp. 166-176. J

194. Fishburn, P. C., "Aspects of One-Stage Voting Rules," Management Science, Vol.21, 1974, pp. 422-427. J

195. Bowman, V. J. and Colantoni, C. S., "Transitive Majority Rule and the Theorem of the Alternative," Opns Res, Vol. 22, 1974, pp. 488-496. J

196. Black, D., "Partial Justification of the Borda Count," Public Choice, Vol XXVIII, Winter 1976, pp. 1-15. J

197. Thrall, R. M., Decision Processes, New York: John Wiley, 1954. K

198. Haith, D. A., Studies in the Analysis of Metropolitan Water Resource Systems, Ithaca, New York: Cornell University, 1971. K

199. Young, H. P., "Social Choice Scoring Functions," SIAM J. of Appl. Math., Vol. 28, June 1975, pp. 824-838. K

200. Davis, J. M., "The Transitivity of Preferences," Behavioral Science, Vol. 3, 1958, pp. 26-33. L

201. Sen, A. K., "Preferences, Votes And The Transitivity Of Majority Decisions," Economic Studies, Vol. 31, April 1964, pp. 163-165. L

202. Inada, K., "The Simple Majority Decision Rule," Econometrica, Vol. 37, 1969, pp. 490-506. L,M

203. Pomeranz, J. E., Weil, R. L., Jr., "Cylical Majority Problem," Commun. of ACM., Vol. 13, No. 4, April 1970, pp. 251-254. L

204. Bowman, V. J. and Colantoni, C. S., "The Extended Concorcet Condition: A Necessary and Sufficient Condition for the Transitivity of Majority Decision," Journal of Mathematical Sociology, Vol. 2, No. 2, July 1972, pp. 267-283. L

205. Jamison, D., Luce, E., "Social Homogeniety and Probability of Intransitive Majority Rule," Journal of Economic Theory, Vol. 5, No. 1, 1972, pp. 79-87. L

206. Rao, S. N., "On A Sufficient Condition For Transitivity of Majority Decision," American Economist, Fall 1972, p. 90. L

207. Kramer, G. H., "On a Class of Equilibrium Conditions for Majority Rule," Econometrica, Vol. 41, No. 2, 1973, pp. 285-297. L

208. Kaneko, M., "Necessary and Sufficient Conditions for Transitivity In Voting Theory," Journal of Economic Theory, Vol. 11, No. 3, 1975, pp. 385-393. L

209. Merchant, D. K. and Rao, M. R., "Majority Decisions and Transitivity: Some Special Cases," Management Science, Vol. 23, Octover 1976, pp. 125-130.

210. McKelvey, R. D., "Intransitivities in Multidimensional Voting Models," Jour. of Economic Theory, Vol. 12, 1976, pp. 472-482. L,P

211. Fishburn, P. C., "Preferences, Summation, and Social Welfare Functions," Management Science, Vol. 16, 1969, pp. 179-186. M

212. Cole, J. D., and Sage, "Multi-person Decision Analysis in Large Scale Systems - Group Decision Making," Franklin Inst. Inl., Vol. 299, April 1975, pp. 245-268. M

213. Fishburn, P. C., "A Strategic Analysis of Nonranked Voting System," SIAM J. On Appl. Math., Vol. 35, 1978, pp. 488-495. M

214. Lazarsfeld, P. F. and Henry, Readings in Mathematical Social Science, Massachusetts Institute of Technology Press, 1966. O

215. Seaver, D. A., Assessment of Group Preferences and Group Uncertainty for Decision Making, Los Angeles: Social Science Research Institute, 1976. O

216. Plott, C. R., "Axiomatic Social Choice Theory: An Overview and Interpretation," Am. J. Political Sci., Vol. 20, 1976. O

217. Sticha, P. J., Tests of Single Coalition and Weighted Majority Theories of Group Decision Making, Michigan University Dissertation, 1977. O

218. Buckley, J. J. and Westen, T. E., "Bargaining Set Theory and Majority Rule," Journal of Conflict Resclution, Vol. 20, No. 3, 1976, pp. 481-496. P

219. Rubinstein, A., "A Note About the 'Nowhere Denseness' of Societies Having An Equilibrium Under Majority Rule," Econometrica, Vol. 47, No. 2, 1979, pp. 511-514. P

220. Tullock, G., "Constitutional Choice and Simple Majority Rule: Reply, " Journal of Political Economy, Vol. 81, No. 2, 1973, pp. 480-484. Q

221. Campbell, C. D. and Tullock, G., "A Measure of the Importance of Cyclical Majorities," Economic Journal, Vol. 75, December 1965, pp. 858-857. S

222. Hinich, M. J., Ledyard, J. L. and Ordeshoo, P. C., "Nonvoting and Existence of Equilibrium Under Majority Rule," Journal of Economic Theory, Vol. 4, No. 2, 1972, pp. 144-153. T

223. Singamsetti, N. R., Majority Rule, Specification and Policy Use of Social Preference Function, New York State University Dissertation, 1974. T

224. Wendell, R. E. and Thorson, S. J., Some Generalizations of Social Decisions Under Majority Rule, Columbus: Ohio State University 1973. T

225. Nitzan, Shmuel, "The Lexicographic Order, Majority Rule and Equilibrium," The Hebrew University of Jerusalem Department of Economics Research Report No. 56, May 1974, pp. 1-17. T

226. Kelly, J. S., "Necessity Conditions in Voting Theory," Journal of Economic Theory, Vol. 8, No. 2, 1974, pp. 149-160. T

227. McKelvey, R. D., Wendell, R. E., "Voting Equilibria in Multi-dimensional Choice Spaces," Mathematics of Operations Research, Vol. 1, No. 2, May 1976, pp. 144-158. T

228. Rader, Trout, "Induced Preferences on Trades When Preferences May Be Intransitive and Incomplete," Econometrica, Vol. 46, No. 1, January 1978, p. 137. T

229. Sengupta, M., "On A Difficulty In The Analysis Of Strategic Voting," Econometrica, Vol. 46, No. 2, March 1978, pp. 331-343. T

230. Shepsle, K. A., "Institutional Arrangements and Equilibrium In Multidimensional Voting Models," American Journal Of Political Science, Vol. 23, No. 1, 1971, pp. 27-59. T

231. Einhorn, H. and Hogarth, R., "Unit Weighting Schemes for Decision Making," Organizational Behavior and Human Performance, Vol. 13, 1975, pp. 171-192. U

232. Whaley, C. P., "Fuzzy Decision-Making," Interface Age, November 1979, pp. 87-91.

233. Dobbins, Edward B. Jr., Research and Development Prioritization - Computer Model, U. S. Army Missile Command, Redstone Arsenal, Alabama 35809, Management Report RN 80-2, April 1980.

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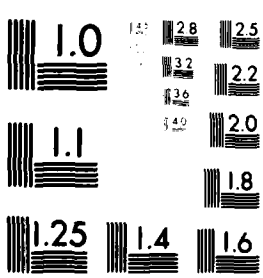

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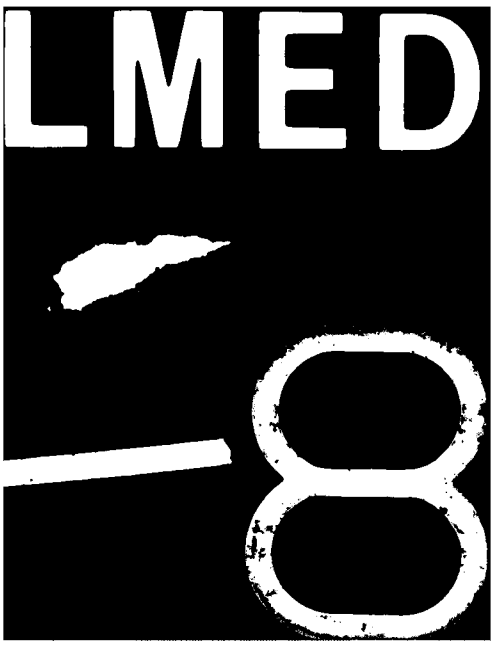
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